Bi-directional Transformations @ UMinho

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Definition

By specifying a transformation on the type-level between A and B we get migration functions between values of A and B and vice-versa.

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User-driven XML schema evolution coupled with document migration. Views of databases where updates to the view induce updates to the database.

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By specifying a transformation on the type-level between A and B we get migration functions between values of A and B and vice-versa.

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Examples

Refinements Going from abstract to concrete.

Lenses Going from concrete to abstract.

Refinements

An abstract type A is mapped to a concrete type B

Representation Injective and total.

Abstraction Surjective and possibly partial.

$$\begin{tikzcd} A \arrow[r, bend left, "\mathit{to}"] \arrow[r, phantom, "\leq"] & B \arrow[l, bend left, "\mathit{from}"] \end{tikzcd}$$

Examples of Refinements

Format evolution

$$\begin{tikzcd} A \arrow[r, bend left, "inject"] \arrow[r, phantom, "\leq"] & A + B \arrow[l, bend left, "isleft"] \end{tikzcd} \qquad \begin{tikzcd} A \arrow[r, bend left, "pairwith(b)"] \arrow[r, phantom, "\leq"] & A \times B \arrow[l, bend left, "project"] \end{tikzcd}$$

Examples of Refinements

Format evolution

$$A \underset{isleft}{\overset{inject}{\rightleftarrows}} A + B \qquad (\leq)$$

$$A \underset{project}{\overset{pairwith(b)}{\rightleftarrows}} A \times B \qquad (\leq)$$

Hierarchical to relational mappings

$$A \rightharpoonup (B \times (C \rightharpoonup D)) \underset{njoin}{\overset{unnjoin}{\rightleftarrows}} (A \rightharpoonup B) \times (A \times C \rightharpoonup D) \qquad (\leq)$$

Composition of Refinements

Sequential composition

$$
\begin{tikzcd}
A \arrow[rr, bend left, "to"] \arrow[rr, phantom, "\leq"] & & B \arrow[ll, bend left, "from"]
\end{tikzcd}
\;\wedge\;
\begin{tikzcd}
B \arrow[rr, bend left, "to'"] \arrow[rr, phantom, "\leq"] & & C \arrow[ll, bend left, "from'"]
\end{tikzcd}
\;\Rightarrow\;
\begin{tikzcd}
A \arrow[rr, bend left, "to' \cdot to"] \arrow[rr, phantom, "\leq"] & & C \arrow[ll, bend left, "from \cdot from'"]
\end{tikzcd}
$$

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Nesting

$$\begin{tikzcd} A \arrow[r, bend left, "to"] \arrow[r, phantom, "\leq"] & B \arrow[l, bend left, "from"] \end{tikzcd} \Rightarrow \begin{tikzcd} \mathsf{F}\, A \arrow[r, bend left, "\mathsf{F}\ to"] \arrow[r, phantom, "\leq"] & \mathsf{F}\, B \arrow[l, bend left, "\mathsf{F}\ from"] \end{tikzcd}$$

Motivation

Two-level Type-level transformation of a data format coupled with the corresponding value-level transformation of data instances.

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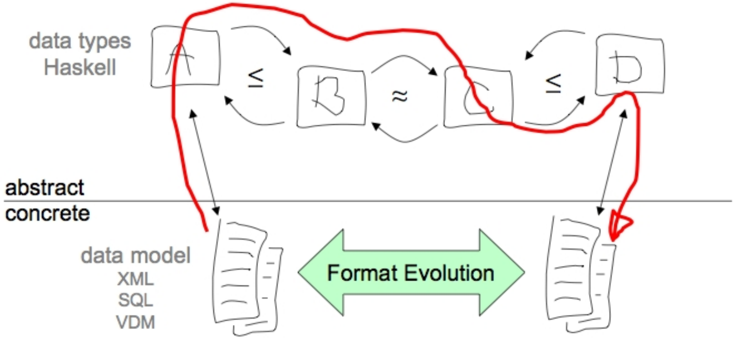
Type-safe Type-checking guarantees that the data migration functions are well-formed with respect to the type-level transformation.

Ingredients

- Concrete data models are abstracted as Haskell data types.
- Type-level transformations are data refinements.
- Strategic programming to compose flexible rewrite systems.



Strategic Programming

- Apply refinement steps ...
  - in what order?
  - how often?
  - at what depth?
  - under which conditions?
- Compose rewrite systems from:
  - basic rewrite rules and
  - combinators for traversal construction.

Combinators

```
(>>>) :: Rule -> Rule -> Rule
(|||) :: Rule -> Rule -> Rule
nop :: Rule
many :: Rule -> Rule
once :: Rule -> Rule
```

Some Implementation Details

Representation of Types

```
data Type a where
    Int  :: Type Int
    List :: Type a -> Type [a]
    Prod :: Type a -> Type b -> Type (a,b)
    ...
```

Some Implementation Details

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Masquerade Changes as Views

```
data Rep a b = Rep {to :: a -> b, from :: b -> a}

data View a where
    View :: Rep a b -> Type b -> View (Type a)
```

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The Type of Rules

```
type Rule = forall a . Type a -> Maybe (View (Type a))
```

Examples of Rules

Refine lists by maps

$$\begin{tikzcd} A^\star \arrow[rr, bend left, "seq2index"] \arrow[rr, phantom, "\leq"] & & \mathbb{N} \rightharpoonup A \arrow[ll, bend left, "list"] \end{tikzcd}$$

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Rule Implementation

```
listmap :: Rule
listmap (List a) = Just (View rep (Map Int a))
    where rep = Rep {to = seq2index, from = list}
listmap _ = Nothing
```

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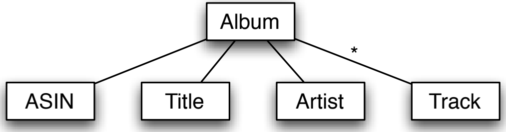
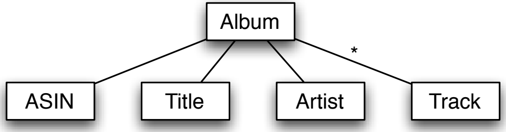
Rewrite system for hierarchical-to-relational mapping

```
flatten :: Rule
flatten = many (once (listmap ||| mapprodmap ||| ...))
```

Example



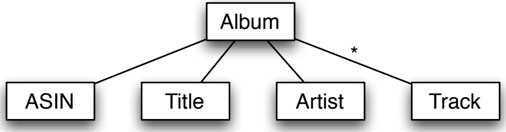
Example



Abstract representation

```
type Album  = (String, (String, (String, [String])))
```

Example



Abstract representation

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```

Converting to a relational database

```
> let (Just vw) = flatten (typeoff :: [Album])
> showType vw
(Map Int (String,(String,String)), Map (Int,Int) String)
```

Motivation

Constrains Rules must take into account constrain information such as primary and secondary keys.

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Front-ends To apply 2LT to real-world examples we need front-ends for popular data description languages, namely XML and SQL.

Type Annotations

Types can be tagged

```
data Type a where
    ...
    Tag :: Tag -> Type a -> Type a

data Tag = Info {name :: Maybe Name,
                 key  :: Maybe Key,
                 refs :: [Key]}
```

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```

Rules manipulate tags

$$({}_kA_r \rightharpoonup (B \times (C \rightharpoonup D)^o))^m$$
$$\cong$$
$$({}_kA_r \rightharpoonup B)^m \times ({}_{\_}A_{kr} \times C \rightharpoonup D)^o$$

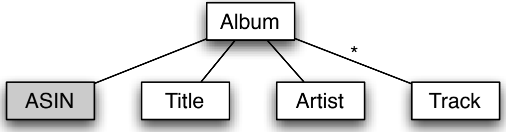
Front-ends

Group them in a class

```
class FrontEnd t v | t -> v, v -> t where
    parsetype  :: t -> Maybe DynType
    printtype  :: Type a -> Maybe t
    parsevalue :: Type a -> v -> Maybe a
    printvalue :: Type a -> a -> Maybe v

instance FrontEnd XSD XML where ...
instance FrontEnd DDL DML where ...
instance FrontEnd VDM VDM where ...
```

Example



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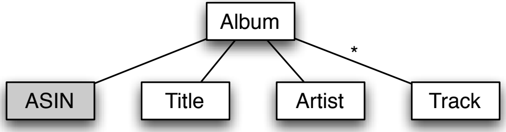

Converting to a relational database

$$({}_1\mathit{String}^{Asin} \rightharpoonup \mathit{String}^{Title} \times \mathit{String}^{Artist})^{Albums} \times$$
$$(\mathit{String}_1^{Asin} \times \mathit{Int} \rightharpoonup \mathit{String}^{Track})^{Tracks}$$

Motivation

Performance The migration functions are very inefficient because of strategic combinators.

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3LT Coupled transformation should encompass not only types and values, but other artifacts like queries and producers.

$$
\begin{tikzcd}
a : A \arrow[rr, Rightarrow, "T"] \arrow[rr, bend left, "to"] \arrow[d, "q"'] & & a' : A' \arrow[ll, bend left, "from"] \arrow[dll, dashed, "q'"] \\
B & &
\end{tikzcd}
$$

Point-free Functional Programming

Some combinators

$$
\begin{array}{l}
id :: A \rightarrow A \\
(\cdot) :: (B \rightarrow C) \rightarrow (A \rightarrow B) \rightarrow (A \rightarrow C) \\
fst :: A \times B \rightarrow A \\
snd :: A \times B \rightarrow B \\
(\vartriangle) :: (A \rightarrow B) \rightarrow (A \rightarrow C) \rightarrow (A \rightarrow B \times C)
\end{array}
$$

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& fst :: A \times B \rightarrow A \\
& snd :: A \times B \rightarrow B \\
& (\triangle) :: (A \rightarrow B) \rightarrow (A \rightarrow C) \rightarrow (A \rightarrow B \times C)
\end{aligned}
$$

Some laws

$$
\begin{aligned}
& f \cdot id = f \wedge id \cdot f = f \\
& f \cdot (g \cdot h) = (f \cdot g) \cdot h \\
& fst \cdot (f \triangle g) = f \wedge snd \cdot (f \triangle g) = g \\
& fst \triangle snd = id \\
& (f \triangle g) \cdot h = (f \cdot h) \triangle (g \cdot h)
\end{aligned}
$$

Rewriting Point-free Expressions

Type-safe representation of functions

```
data PF f where
    Id    :: PF (a -> a)
    Fst   :: PF ((a,b) -> a)
    (/\)  :: PF (a -> b) -> PF (a -> c) -> PF (a -> (b,c))
    ...
```

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Another strategic rewrite system

```
type RULE = forall f . Type f -> PF f -> RewriteM (PF f)
```

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Migration functions represented in point-free

```
data Rep a b = Rep {to :: PF (a -> b), from :: PF (b -> a)}

data View a where
    View :: Rep a b -> Type b -> View (Type a)

type Rule = forall a . Type a -> Maybe (View (Type a))
```

Example

Type refinement

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type AlbumsDB = (Map Int (String,(String,String)),
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```

Query migration

```
getArtists :: [Album] -> [String]
getArtists = List.map (fst . snd . snd)

getArtists' :: AlbumsDB -> [String]
getArtists' = elems . Map.map (snd . snd) . fst
```

Motivation

Partiality To deal correctly with query migration the abstraction function must be explicitly partial.

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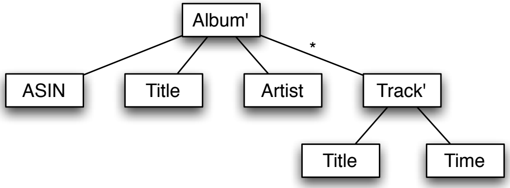
Generics How to deal with structure-shy languages like XPath? Does `//Title` returns the expected results after refining tracks to have both title and duration?



Composition With Partiality

Sequential composition

$$A+1 \xleftarrow{from} B \ \wedge\ B+1 \xleftarrow{from'} C$$
$$\Rightarrow$$
$$A+1 \xleftarrow{id \triangledown inr} (A+1)+1 \xleftarrow{from+id} B+1 \xleftarrow{from'} C$$

Nesting

$$A+1 \xleftarrow{from} B$$
$$\Rightarrow$$
$$[A]+1 \xleftarrow{inl \cdot concat} [[A]] \xleftarrow{map\ (wrap \triangledown nil)} [A+1] \xleftarrow{map\ \ from} [B]$$

Structure-Shy Languages

SYB

$$
\begin{aligned}
(\triangleright) &:: \mathrm{T} \to \mathrm{T} \to \mathrm{T} \\
mapT &:: \mathrm{T} \to \mathrm{T} \\
mkT_A &:: (A \to A) \to \mathrm{T} \\
apT_A &:: \mathrm{T} \to (A \to A) \\
mapQ &:: \mathrm{Q}\ R \to \mathrm{Q}\ R \\
mkQ_A &:: (A \to R) \to \mathrm{Q}\ R \\
apQ_A &:: \mathrm{Q}\ R \to (A \to R)
\end{aligned}
$$

XPath

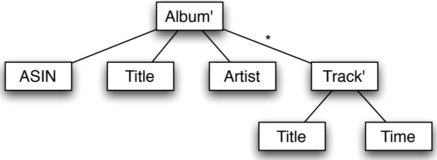
$$
\begin{aligned}
child &:: \mathrm{Q}\ [*] \\
descorself &:: \mathrm{Q}\ [*] \\
name &:: String \to \mathrm{Q}[*] \\
(/) &:: \mathrm{Q}\ [*] \to \mathrm{Q}\ R \to \mathrm{Q}\ R
\end{aligned}
$$

Specializing Structure-Shy Programs

Some Laws

$$
\begin{aligned}
& apT_A\ (f \triangleright g) = apT_A\ f \cdot apT_A\ g \\
& apT_A\ (mkT_A\ f) = f \\
& apT_A\ (mkT_B\ f) = id, \mathbf{if} A \neq B \\
& apT_(A \times B)\ (mapT\ \ f) = apT_A\ f \times apT_B\ f \\
& apQ_A\ (mkQ_A\ f) = f \\
& apQ_A\ (mkQ_B\ f) = zero, \mathbf{if} A \neq B \\
& apQ_(A \times B)\ (mapQ\ \ f) = plus \cdot (apQ_A\ f \times apQ_B\ f) \\
& descendant\ = everything\ \ child \\
& apQ_A\ (f / g) = fold \cdot map\ (apQ_*\ g) \cdot apQ_A\ f
\end{aligned}
$$

Example



Specialization of //Title to Album'

```
cat . (map (mkAny . title) /\
       concat . map (map (mkAny . track_title) . tracks))
    where title  = fst . snd . unAlbum'
          tracks = snd . snd . snd . unAlbum'
          track_title = fst . unTrack'
```

Specialization of //Title to Album composed with abstraction

```
map (mkAny . title)
    where title  = fst . snd . unAlbum'
```

Lenses

A concrete type A is abstracted into a view B

$$\begin{tikzcd} A \arrow[r, bend left, "get"] \arrow[r, phantom, "\geq"] & B \times A \arrow[l, bend left, "putback"] \end{tikzcd}$$

Properties

Acceptability *putback* must be injective on first argument.

$$get \cdot putback \sqsubseteq fst$$

Stability If the target does not change, neither should the source.

$$putback \cdot (get \vartriangle id) \sqsubseteq id$$

Example of Lenses

Drop field

$$\begin{tikzcd} A \times B \arrow[rr, bend left, "fst"] \arrow[rr, phantom, "\geq"] & & A \times A \times B \arrow[ll, bend left, "id \times snd"] \end{tikzcd}$$

Composition of Lens

Sequential composition

$$
\begin{tikzcd}
A \arrow[rr, bend left, "get"] \arrow[rr, phantom, "\geq"] & & B \times A \arrow[ll, bend left, "putback"]
\end{tikzcd}
\quad \wedge \quad
\begin{tikzcd}
B \arrow[rr, bend left, "get'"] \arrow[rr, phantom, "\geq"] & & C \times B \arrow[ll, bend left, "putback'"]
\end{tikzcd}
$$

$$\Rightarrow$$

$$
\begin{tikzcd}
A \arrow[rr, bend left, "get' \cdot get"] \arrow[rr, phantom, "\geq"] & & C \times A \arrow[ll, bend left]
\end{tikzcd}
$$

$$A \xleftarrow{putback} B \times A \xleftarrow{putback' \times id} (C \times B) \times A \xleftarrow{(id \times get) \triangle id} C \times A$$

The Future of 2LT

- We want a single version of 2LT but...
  - Recursion and tags are not well supported in the `rule` release.
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  - Front-ends to other query languages, namely SQL.
  - Support for mutually recursive types.

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- And we would love to support lenses!