Gielis Transformations in mathematics and the natural sciences

Johan Gielis, Diego Caratelli and Tom Gerats

In the development of mathematics and natural sciences geometry plays a fundamental and pivotal role. Geometry deals both with the descriptive and the metrical aspects of natural shapes and phenomena. Euclidean geometry still holds a crucial place, for the study of both intrinsic and extrinsic measures in Riemannian geometries, where the generalized Pythagorean Theorem is key. Gielis transformations are another simple generalization of the Pythagorean Theorem, based only on pure numbers. They introduce anisotropy from the start, using specific unit circles, in the spirit of Minkowski and Riemann-Finsler geometry, as generalized intrinsic and extrinsic lengths in submanifolds. The transformation defines measures and unit elements specific to the shape, but in the end it is just Euclidean geometry.

Global anisotropies or (quasi-) periodic local deviations from isotropy or Euclidean perfection in many forms that occur in nature can be effectively dealt with by applying Gielis transformations to the basic forms that show up in Euclidean geometry, e.g. circle and spiral. Anisotropic versions of the classical constant mean curvature and minimal surfaces have been developed and in mathematical physics it has led to developing analytical solutions to a variety of boundary value problems.

In the modeling of plants it can be used to model a wide variety of plant shapes, and even a complete plant (and its evolution) might be expressed as one complete equation. Here we focus on fusion, one of the most pronounced key innovations in the evolution of plants. We present a geometric model for shape and fusion in the corolla of Asclepiads. Examples demonstrate how fusion of petals creates stable centers, a prerequisite for the formation of complex pollination structures specific to Asclepiads, via congenital and postgenital fusion events, with the formation of de novo organs. The development of the corolla reduces to simple inequalities from the MATHS-box. The formation of stable centers and of bell and tubular shapes in flowers are immediate and logical consequences of the shape. Our model shows that any study on flowers, especially in evo-devo perspective should be performed within the wider framework of polymery and oligomery and of fusion and synorganisation.