# Coalgebraic Logics and Synthesis for Mealy Machines

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Coalgebra Day

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# Motivation

## Design of Sequential systems

- (Binary) Mealy machines $\leftrightarrow$ digital circuits
- There is no formal way of specifying Mealy machines
- Typically they are “defined” in a natural language, such as English

**Source of ambiguities**

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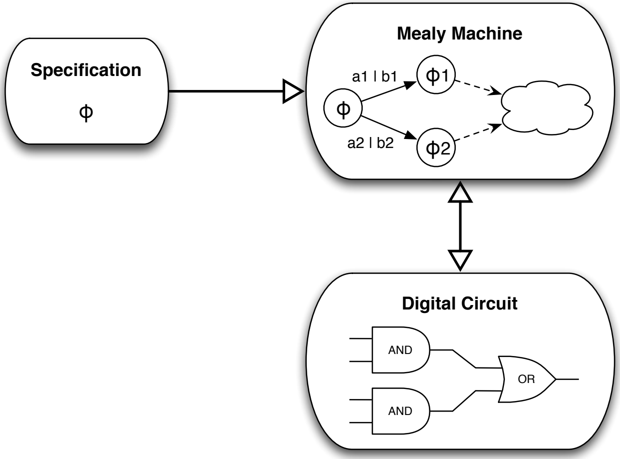

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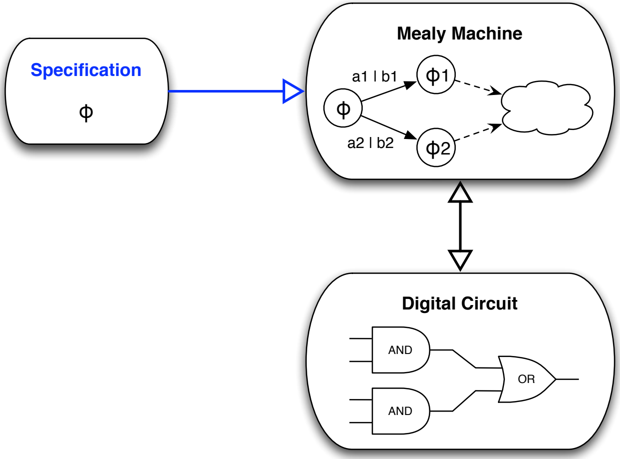

# But first...

## What do we mean by **Binary Mealy Machine?**

- Mealy machines = **Deterministic Mealy machines**
- Mealy machine = set of states $S$ + transition function $f$

$$
\begin{array}{lcl}
f & : & S \to (B \times S)^A \\
f(s)(a) & = & < b, s' >
\end{array}
$$

$$s \xrightarrow{a|b} s'$$

  $A$ is the input alphabet and $B$ is the output alphabet
- Binary Mealy machines : $A = 2$ and

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B = \mathcal{B} = \begin{tikzcd}[column sep=small] & \top \arrow[dl, dash] \arrow[dr, dash] & \\ 1 \arrow[dr, dash] & & 0 \arrow[dl, dash] \\ & \bot & \end{tikzcd}
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# Information order

$$\mathcal{B} = \begin{tikzcd} & \top \arrow[dl, dash] \arrow[dr, dash] & \\ 1 \arrow[dr, dash] & & 0 \arrow[dl, dash] \\ & \bot & \end{tikzcd}$$

- $\top$ – *abstraction* (under-specification)
- $\bot$ – *inconsistency* (over-specification)
- 0 and 1 – concrete output values.

# Mealy automata are coalgebras

## Observation:

A Mealy machine is a coalgebra of the functor

$$\begin{aligned} M &: Set \to Set \\ M(X) &= (B \times X)^A \end{aligned}$$

## (Almost) for free:

- Notion of (bi)simulation : equivalence between states, minimization
- Semantics in terms of final coalgebra – causal functions
- Specification language for Mealy machines

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**Remark** Simple but expressive language: Every finite Mealy machine corresponds to a finite formula in our language.

# Simple properties

- Output 0 at each input of 1

$$1 \downarrow 0$$

- Output 0 at each input of two consecutive 1's

$$\nu x.(1(1 \downarrow 0 \wedge 1(x) \wedge 0(x)) \wedge 0(x))$$

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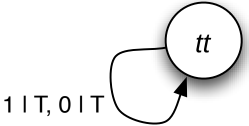

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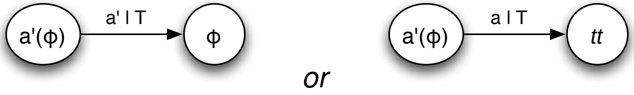

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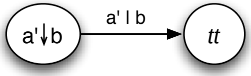


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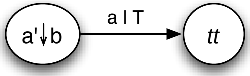

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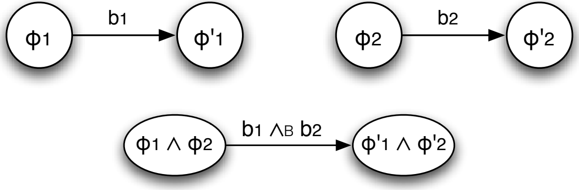

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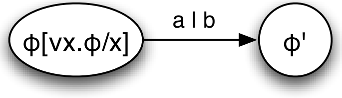

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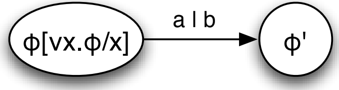


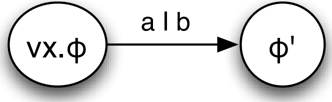

# Why a coalgebra structure?

## Two advantages

1. Semantics

$$\begin{tikzcd} L \arrow[r, "[\![\cdot]\!]"] \arrow[d, "\lambda"'] & \Gamma \arrow[d, "\gamma"] \\ (B \times L)^A \arrow[r] & (B \times \Gamma)^A \end{tikzcd}$$

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$$s \models \phi \Leftrightarrow s \lesssim \phi$$

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# Logic is expressive

**Theorem**

1. *For all states $s$, $s'$ of a Mealy machine $(S, f)$,*

$$s \sim s' \text{ iff } \forall_{\phi \in L}. s \models \phi \Leftrightarrow s' \models \phi.$$

2. *If $S$ is finite then there exists for any $s \in S$ a formula $\phi_s$ such that $s \sim \phi$.*

We also want:

For every formula $\phi$ construct a **finite** Mealy machine $(S, f)$ such that $\exists_{s \in S}\, s \sim \phi$.

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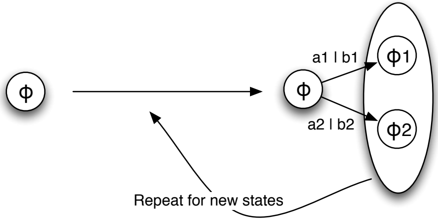
# But. . .

Easy answer: Apply $\lambda$ repeatedly!




- $\lambda$ will not deliver a finite automata.

$$\phi = \nu x.1(x \wedge (\nu y.1(y)))$$

$$\lambda(\phi)(1) =< \top, \phi \wedge (\nu y.1(y)) >$$
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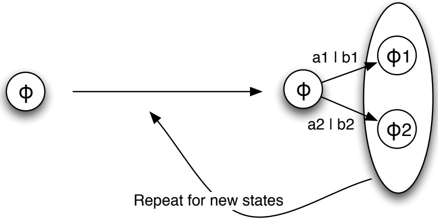


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# Normalization

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norm(tt) & = & tt \\
norm(a(\phi)) & = & a(norm(\phi)) \\
norm(a \downarrow b) & = & a \downarrow b \\
norm(\phi_1 \wedge \phi_2) & = & conj(rem(flatten(norm(\phi_1) \wedge norm(\phi_2)))) \\
norm(\nu x.\phi) & = & \nu x.norm(\phi)
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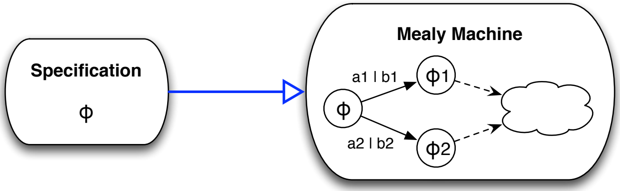

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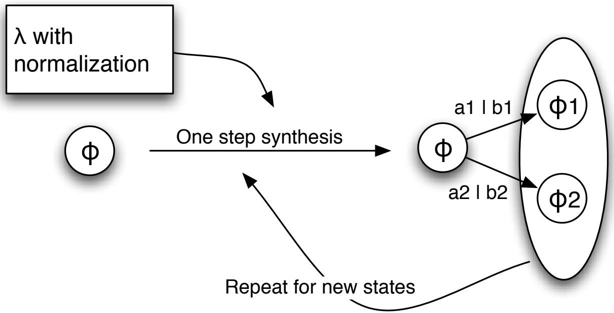
λ with
normalization
φ
One step synthesis
φ
a1 | b1
φ1
a2 | b2
φ2
Repeat for new states

# One-step synthesis

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\delta & : & L \to (B \times L)^A \\
\delta(tt) & = & < \top, tt > \\
\delta(a'(\phi))(a) & = & \begin{cases} < \top, norm(\phi) > & a = a' \\ < \top, tt > & otherwise \end{cases} \\
\delta(a' \downarrow b)(a) & = & \begin{cases} < b, tt > & a = a' \\ < \top, tt > & otherwise \end{cases} \\
\delta(\phi_1 \wedge \phi_2)(a) & = & \delta(\phi_1)(a) \sqcap \delta(\phi_2)(a) \\
\delta(\nu x.\phi)(a) & = & < b, norm(\phi') > \\
 & & where < b, \phi' >= \delta(\phi[\nu x.\phi/x])(a)
\end{array}
$$

$$
< b_1, \phi_1 > \quad \sqcap \quad < b_2, \phi_2 > = < b_1 \wedge_B b_2, norm(\phi_1 \wedge \phi_2) >
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# Examples

- $\phi = 1 \downarrow 0 \wedge (\nu x.1(x))$

$$\begin{array}{rcl} \delta(\phi)(0) & = & < \top, tt > \\ \delta(\phi)(1) & = & \delta(1 \downarrow 0)(1) \sqcap \delta(\nu x.1(x))(1) \\ & = & < 0, tt > \sqcap < \top_B, \nu x.1(x) > \\ & = & < 0, \nu x.1(x) > \end{array}$$

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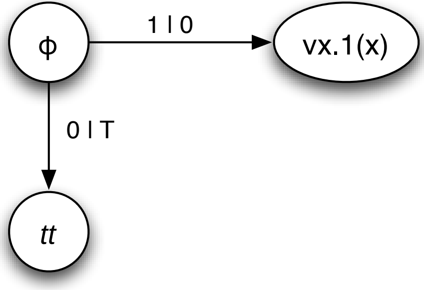

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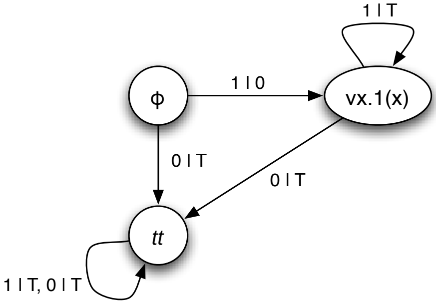

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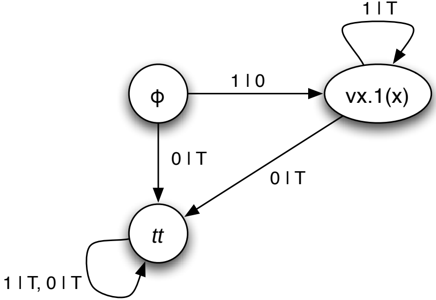


**Remark:** Not minimal.

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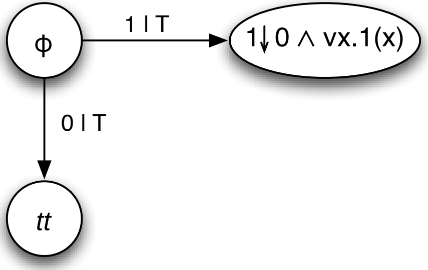

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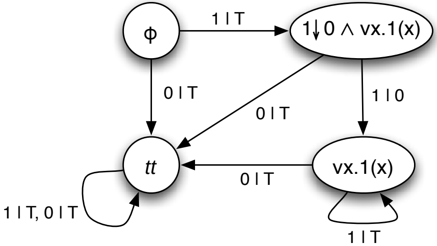
$$\begin{array}{lcl} \delta(\phi_2)(0) & = & <\top, tt> \\ \delta(\phi_2)(1) & = & \delta(1(1\downarrow 0))(1) \sqcap \delta(\nu x.1(x))(1) \\ & = & <\top, 1\downarrow 0> \sqcap <\top, \nu x.1(x)> \\ & = & <\top, 1\downarrow 0 \wedge (\nu x.1(x))> \end{array}$$



# Examples

- $\phi_3 = \nu x.1(x \wedge (\nu y.1(y) \wedge 1 \downarrow 0))$

$$\begin{aligned} \delta(\phi_3)(0) &= <\top, tt> \\ \delta(\phi_3)(1) &= <\top, \phi_3 \wedge (\nu y.1(y) \wedge 1 \downarrow 0)> \end{aligned}$$

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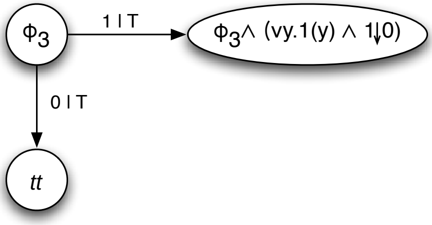
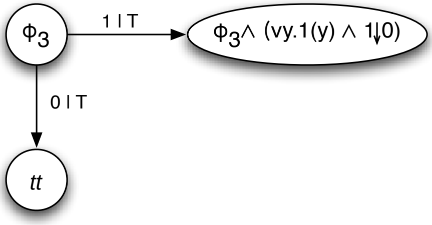


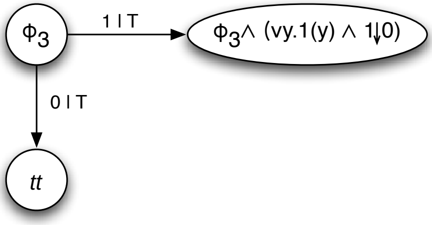
# Examples (cont.)

$$\delta(\phi_3 \wedge (\nu y.1(y) \wedge 1 \downarrow 0))(1)$$



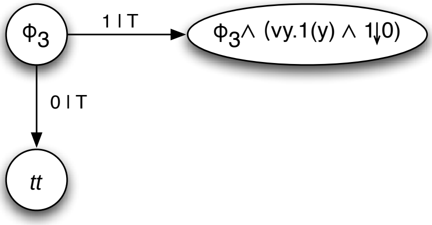
# Examples (cont.)

$$
\begin{aligned}
& \delta(\phi_3 \wedge (\nu y.1(y) \wedge 1 \downarrow 0))(1) \\
= \;& \delta(\phi_3)(1) \sqcap \delta(\nu y.1(y) \wedge 1 \downarrow 0)(1)
\end{aligned}
$$



# Examples (cont.)

$$\begin{array}{rl} & \delta(\phi_3 \wedge (\nu y.1(y) \wedge 1 \downarrow 0))(1) \\ = & \delta(\phi_3)(1) \sqcap \delta(\nu y.1(y) \wedge 1 \downarrow 0)(1) \\ = & < \top, \phi \wedge (\nu y.1(y) \wedge 1 \downarrow 0) > \sqcap < 0, \nu y.1(y) \wedge 1 \downarrow 0 > \end{array}$$



$$
\begin{aligned}
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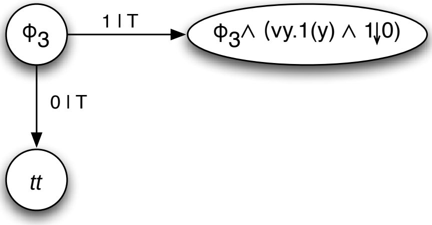
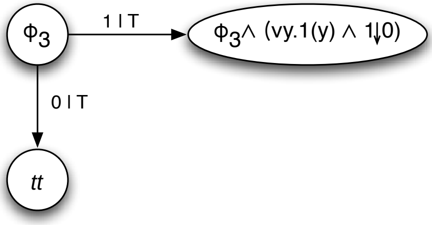
= \; & \delta(\phi_3)(1) \sqcap \delta(\nu y.1(y) \wedge 1 \downarrow 0)(1) \\
= \; & < \top, \phi \wedge (\nu y.1(y) \wedge 1 \downarrow 0) > \sqcap < 0, \nu y.1(y) \wedge 1 \downarrow 0 > \\
= \; & < 0, \mathit{norm}(\phi_3 \wedge (\nu y.1(y) \wedge 1 \downarrow 0) \wedge (\nu y.1(y) \wedge 1 \downarrow 0)) >
\end{aligned}
$$



# Examples (cont.)

$$\begin{array}{rl} & \delta(\phi_3 \wedge (\nu y.1(y) \wedge 1 \downarrow 0))(1) \\ = & \delta(\phi_3)(1) \sqcap \delta(\nu y.1(y) \wedge 1 \downarrow 0)(1) \\ = & < \top, \phi \wedge (\nu y.1(y) \wedge 1 \downarrow 0) > \sqcap < 0, \nu y.1(y) \wedge 1 \downarrow 0 > \\ = & < 0, norm(\phi_3 \wedge (\nu y.1(y) \wedge 1 \downarrow 0) \wedge (\nu y.1(y) \wedge 1 \downarrow 0)) > \\ = & < 0, \phi_3 \wedge (\nu y.1(y) \wedge 1 \downarrow 0) > \end{array}$$



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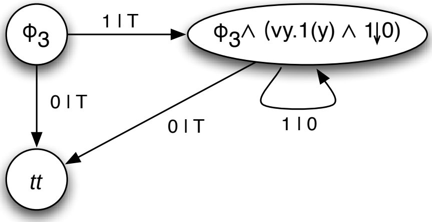

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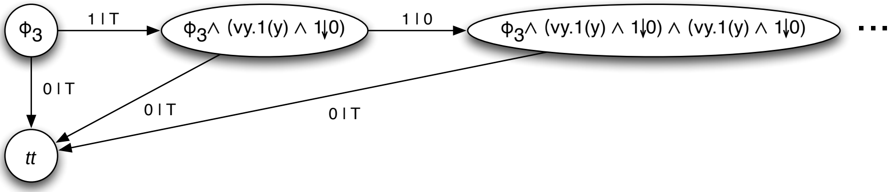
$$\begin{array}{rl} & \delta(\phi_3 \wedge (\nu y.1(y) \wedge 1 \downarrow 0))(1) \\ = & \delta(\phi_3)(1) \sqcap \delta(\nu y.1(y) \wedge 1 \downarrow 0)(1) \\ = & < \top, \phi \wedge (\nu y.1(y) \wedge 1 \downarrow 0) > \sqcap < 0, \nu y.1(y) \wedge 1 \downarrow 0 > \\ = & < 0, norm(\phi_3 \wedge (\nu y.1(y) \wedge 1 \downarrow 0) \wedge (\nu y.1(y) \wedge 1 \downarrow 0)) > \\ = & < 0, \phi_3 \wedge (\nu y.1(y) \wedge 1 \downarrow 0) > \end{array}$$



# Conclusions

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- Coalgebraic approach : bisimulation and logics
- New logic for Mealy machines
- Synthesis algorithm that produces a finite machine

## Future Work

- The logic has exactly the operators needed to represent all Mealy machines
- Similar to regular expressions for deterministic automata
- Can we generalize this approach to other types of coalgebras?

**Towards Regular expressions for polynomial coalgebras**

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