



CSL 2016

Marseille, France



Coalgebraic learning

Alexandra Silva

The top portion of the slide features a complex, abstract network of red and yellow lines and nodes, resembling a data visualization or a neural network structure. The lines are thin and numerous, creating a dense web of connections. The nodes are small circles in red and yellow, scattered throughout the network. The overall color palette is dominated by reds and oranges, with some yellow highlights.

Automata learning

Learning



Automata learning

Active learning

Passive learning



Automata learning

Active learning

Dana Angluin. Learning regular sets from queries and counterexamples. *Inf. Comput.*, 75(2):87–106, 1987.



Automata learning

Active learning

Dana Angluin. Learning regular sets

L^* - algorithm

J. Res. Inf. Comput., 75(2):87–106, 1987.

Automata learning

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Deterministic automata — only **simple** regular languages

Automata learning

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Deterministic automata — only **simple** regular languages

simple is
beautiful.

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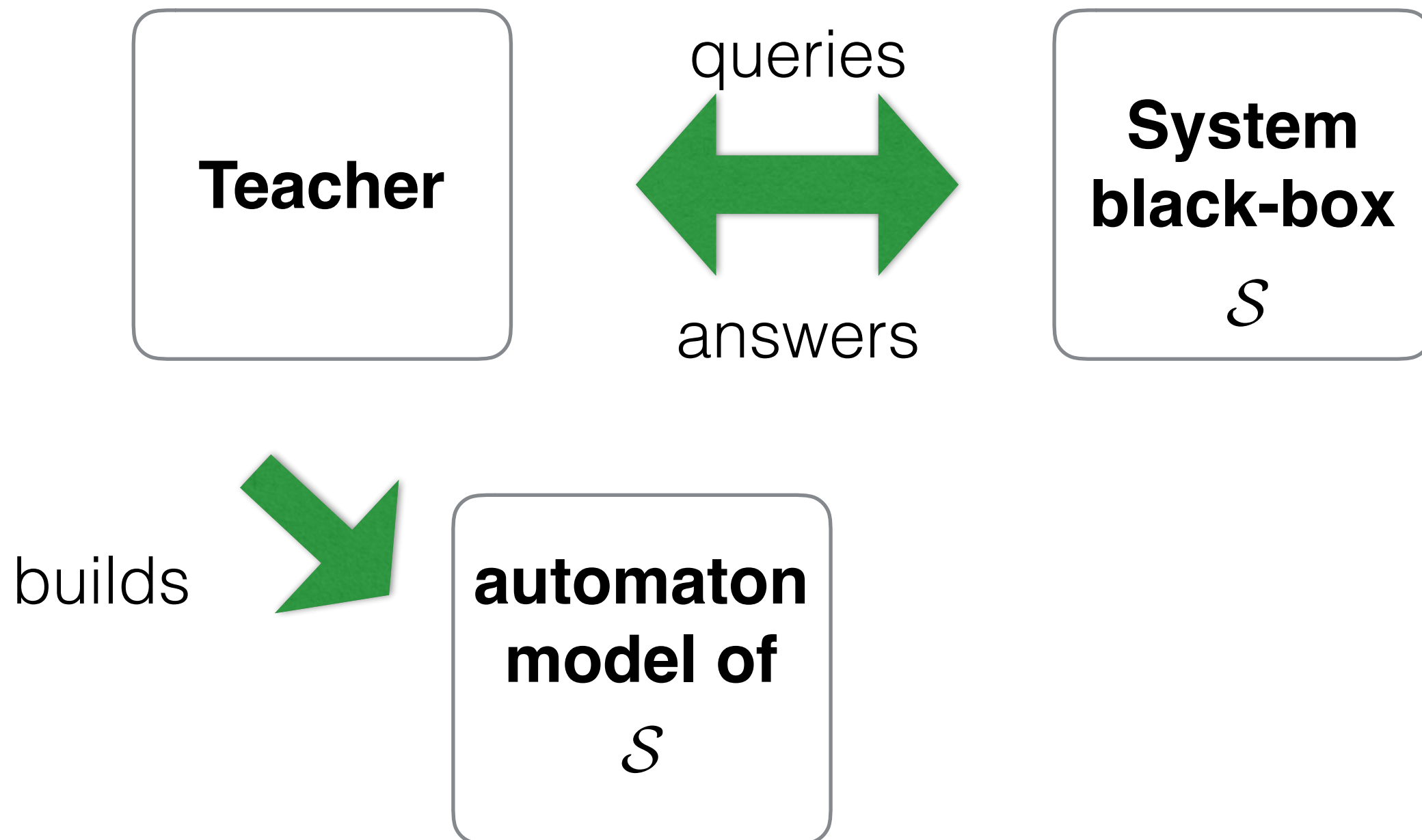
Deterministic automata — only **simple** regular languages

simple is
beautiful.

&

POWERFUL

L^* , by example



L^* , by example

Teacher queries

Membership queries $w \in \mathcal{L}?$

Equivalence queries $\mathcal{L}(H) = \mathcal{L}?$

Yes :-)

No :-(+ counter-example

L^* , by example

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Yes :-)

No :- (+ counter-example

Observation table

| | | E | | |
|--------------------|------------|------------|-----|------|
| | | ϵ | a | aa |
| $S \cup S \cdot A$ | ϵ | 0 | 0 | 1 |
| | a | 0 | 1 | 0 |
| | b | 0 | 0 | 0 |

$S, E \subseteq A^*$

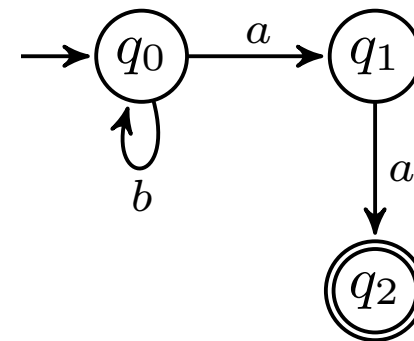
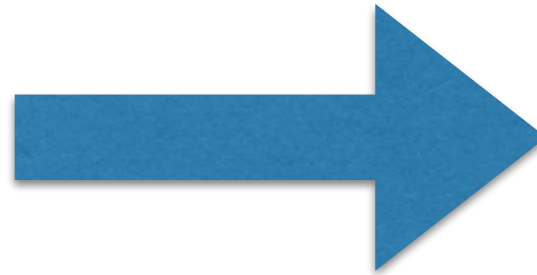
$row: S \cup S \cdot A \rightarrow 2^E$

$row(u)(v) = 1 \iff uv \in \mathcal{L}$

L^* , by example

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| ϵ | 0 | 0 |
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| aab | 0 | 0 |

(S, E)

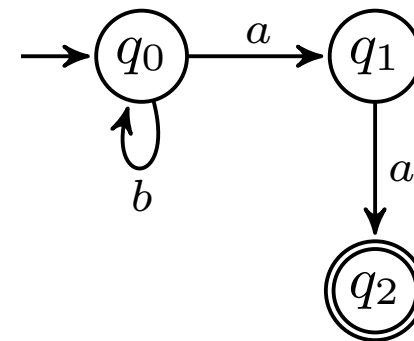
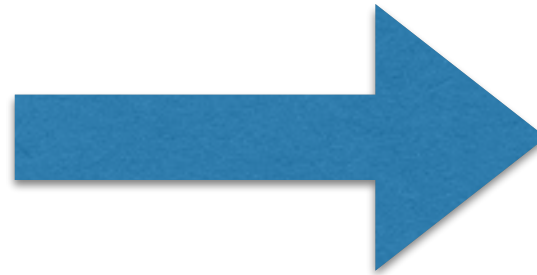


(Q, q_0, F, δ)

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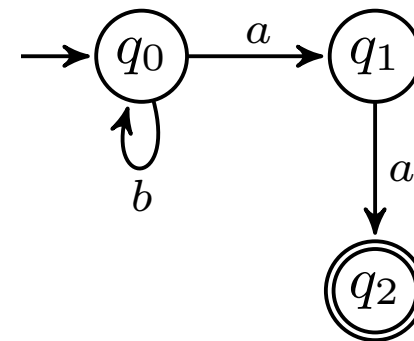
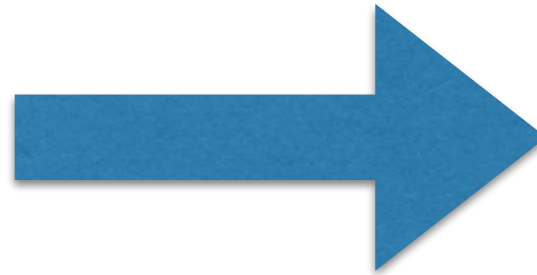
(Q, q_0, F, δ)

- $Q = \{row(s) \mid s \in S\}$ is a finite set of states;
- $F = \{row(s) \mid s \in S, row(s)(\epsilon) = 1\} \subseteq Q$ is the set of final states;
- $q_0 = row(\epsilon)$ is the initial state;
- $\delta: Q \times A \rightarrow Q$ is the transition function given by $\delta(row(s), a) = row(sa)$.

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(S, E)



(Q, q_0, F, δ)

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Why is this well-defined?

Table properties

$$row: S \cup S \cdot A \rightarrow 2^E$$

closed

$$\forall t \in S \cdot A \quad \exists s \in S \quad row(t) = row(s).$$

consistent

$$\forall s_1, s_2 \text{ s.t. } row(s_1) = row(s_2) \Rightarrow \forall a \in A \quad row(s_1 a) = row(s_2 a).$$

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$row(sa)$ is a state

$$\delta(row(s), a) = row(sa) \quad \text{well-defined}$$

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choice of $row(s)$ as representative is irrelevant
 $\delta(row(s), a) = row(sa)$ well-defined

L^* , by example

L^* LEARNER

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$$\mathcal{L}_n = \{ww \mid w \in A^*, |w| = n\}$$

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14 return  $M(S, E)$ 

```

| | ϵ |
|------------|------------|
| ϵ | 0 |
| a | 0 |
| b | 0 |

$$\mathcal{A}_1 = \rightarrow (q_0) \xrightarrow{a/b}$$

$$q_0 = row(\epsilon) = \{\epsilon \mapsto 0\}$$

$$t = aa$$

| | ϵ |
|------------|------------|
| ϵ | 0 |
| a | 0 |
| aa | 1 |
| b | 0 |
| ab | 0 |
| aaa | 0 |
| aab | 0 |

$$\mathcal{L}_n = \{ww \mid w \in A^*, |w| = n\}$$

$$\mathcal{L}_1 = \{\bar{a}a, \bar{b}b\}$$

L*, by example

L* LEARNER

```

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| | ϵ |
|------------|------------|
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 $t = aa$

| | ϵ | | ϵ | a |
|------------|------------|--|------------|-----|
| ϵ | 0 | | 0 | 0 |
| a | 0 | | 0 | 1 |
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| | ϵ | a |
|------------|------------|-----|
| ϵ | 0 | 0 |
| a | 0 | 1 |
| aa | 1 | 0 |
| b | 0 | 0 |
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9       $E \leftarrow E \cup \{ae\}$ 
10   Make the conjecture  $M(S, E)$ 
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```

| | ϵ |
|------------|------------|
| ϵ | 0 |
| a | 0 |
| b | 0 |

$$\mathcal{A}_1 = \rightarrow (q_0) \xrightarrow{a/b} q_0$$

$$q_0 = row(\epsilon) = \{\epsilon \mapsto 0\}$$

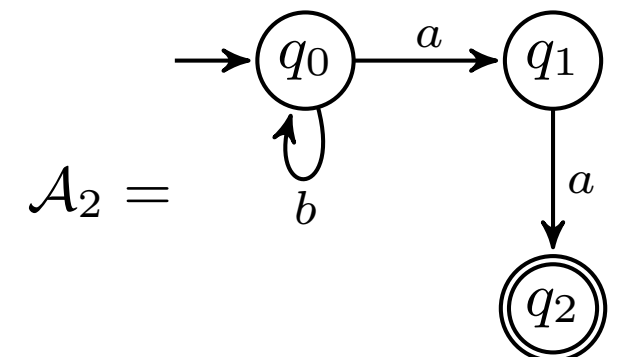
$$t = aa$$

| | ϵ |
|------------|------------|
| ϵ | 0 |
| a | 0 |
| aa | 1 |
| b | 0 |
| ab | 0 |
| aaa | 0 |
| aab | 0 |

| | ϵ | a |
|------------|------------|-----|
| ϵ | 0 | 0 |
| a | 0 | 1 |
| aa | 1 | 0 |
| b | 0 | 0 |
| ab | 0 | 0 |
| aaa | 0 | 0 |
| aab | 0 | 0 |

$$\mathcal{L}_n = \{ww \mid w \in A^*, |w| = n\}$$

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L*, by example

L* LEARNER

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| | ϵ |
|------------|------------|
| ϵ | 0 |
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 $q_0 = row(\epsilon) = \{\epsilon \mapsto 0\}$

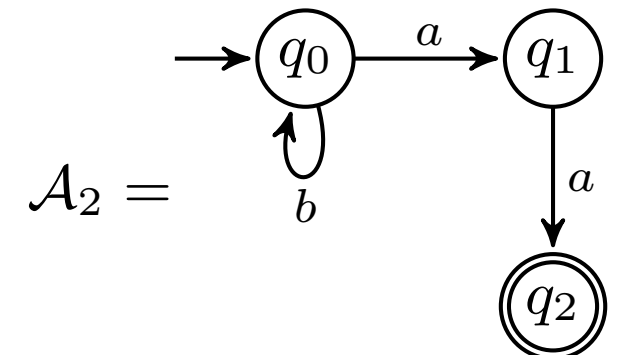
$t = aa$

| | ϵ |
|------------|------------|
| ϵ | 0 |
| a | 0 |
| aa | 1 |
| b | 0 |
| ab | 0 |
| aaa | 0 |
| aab | 0 |

| | ϵ | a |
|------------|------------|-----|
| ϵ | 0 | 0 |
| a | 0 | 1 |
| aa | 1 | 0 |
| b | 0 | 0 |
| ab | 0 | 0 |
| aaa | 0 | 0 |
| aab | 0 | 0 |

$$\mathcal{L}_n = \{ww \mid w \in A^*, |w| = n\}$$

$$\mathcal{L}_1 = \{aa, bb\}$$



L*, by example

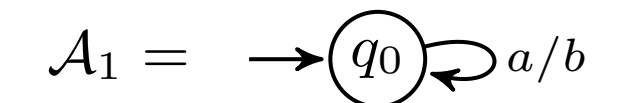
L* LEARNER

```

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| | ϵ |
|------------|------------|
| ϵ | 0 |
| a | 0 |
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$$q_0 = row(\epsilon) = \{\epsilon \mapsto 0\}$$

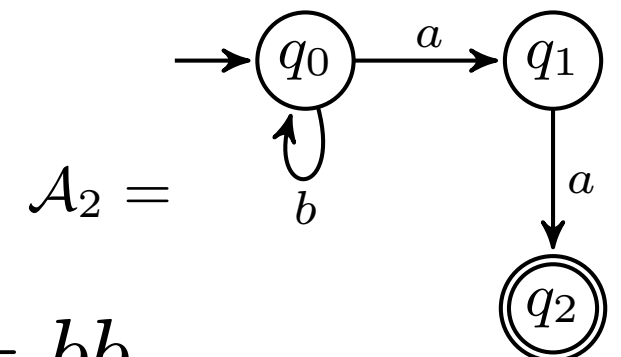
$$t = aa$$

| | ϵ |
|------------|------------|
| ϵ | 0 |
| a | 0 |
| aa | 1 |
| b | 0 |
| ab | 0 |
| aaa | 0 |
| aab | 0 |

| | ϵ | a |
|------------|------------|-----|
| ϵ | 0 | 0 |
| a | 0 | 1 |
| aa | 1 | 0 |
| b | 0 | 0 |
| ab | 0 | 0 |
| aaa | 0 | 0 |
| aab | 0 | 0 |

$$\mathcal{L}_n = \{ww \mid w \in A^*, |w| = n\}$$

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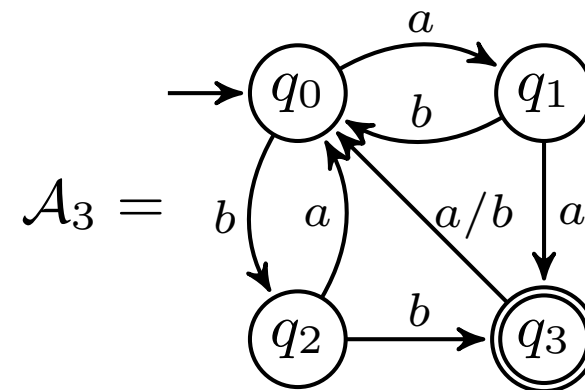


$$t = bb$$

L^* , by example

| | ϵ | a |
|------------|------------|-----|
| ϵ | 0 | 0 |
| a | 0 | 1 |
| aa | 1 | 0 |
| b | 0 | 0 |
| bb | 1 | 0 |
| ab | 0 | 0 |
| aaa | 0 | 0 |
| aab | 0 | 0 |
| ba | 0 | 0 |
| bba | 0 | 0 |
| bbb | 0 | 0 |

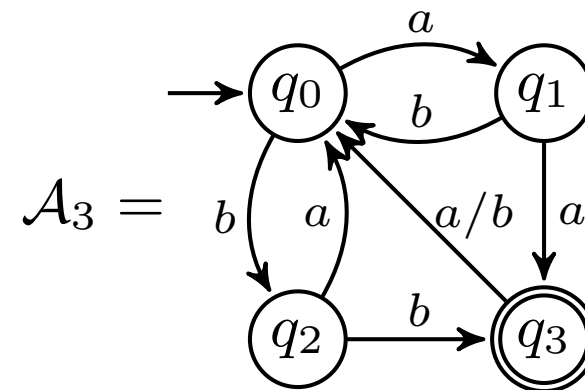
| | ϵ | a | b |
|------------|------------|-----|-----|
| ϵ | 0 | 0 | 0 |
| a | 0 | 1 | 0 |
| aa | 1 | 0 | 0 |
| b | 0 | 0 | 1 |
| bb | 1 | 0 | 0 |
| ab | 0 | 0 | 0 |
| aaa | 0 | 0 | 0 |
| aab | 0 | 0 | 0 |
| ba | 0 | 0 | 0 |
| bba | 0 | 0 | 0 |
| bbb | 0 | 0 | 0 |



L^* , by example

| | ϵ | a |
|------------|------------|-----|
| ϵ | 0 | 0 |
| a | 0 | 1 |
| aa | 1 | 0 |
| b | 0 | 0 |
| bb | 1 | 0 |
| ab | 0 | 0 |
| aaa | 0 | 0 |
| aab | 0 | 0 |
| ba | 0 | 0 |
| bba | 0 | 0 |
| bbb | 0 | 0 |

| | ϵ | a | b |
|------------|------------|-----|-----|
| ϵ | 0 | 0 | 0 |
| a | 0 | 1 | 0 |
| aa | 1 | 0 | 0 |
| b | 0 | 0 | 1 |
| bb | 1 | 0 | 0 |
| ab | 0 | 0 | 0 |
| aaa | 0 | 0 | 0 |
| aab | 0 | 0 | 0 |
| ba | 0 | 0 | 0 |
| bba | 0 | 0 | 0 |
| bbb | 0 | 0 | 0 |

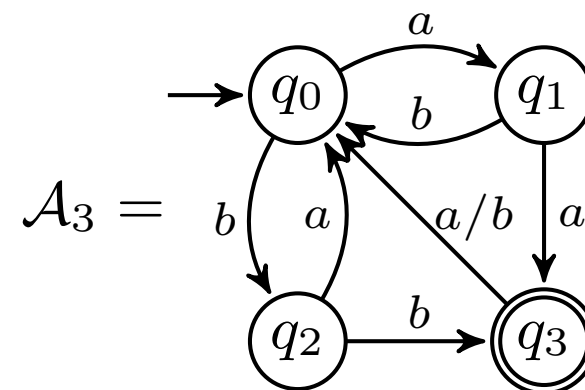


$t = babb$

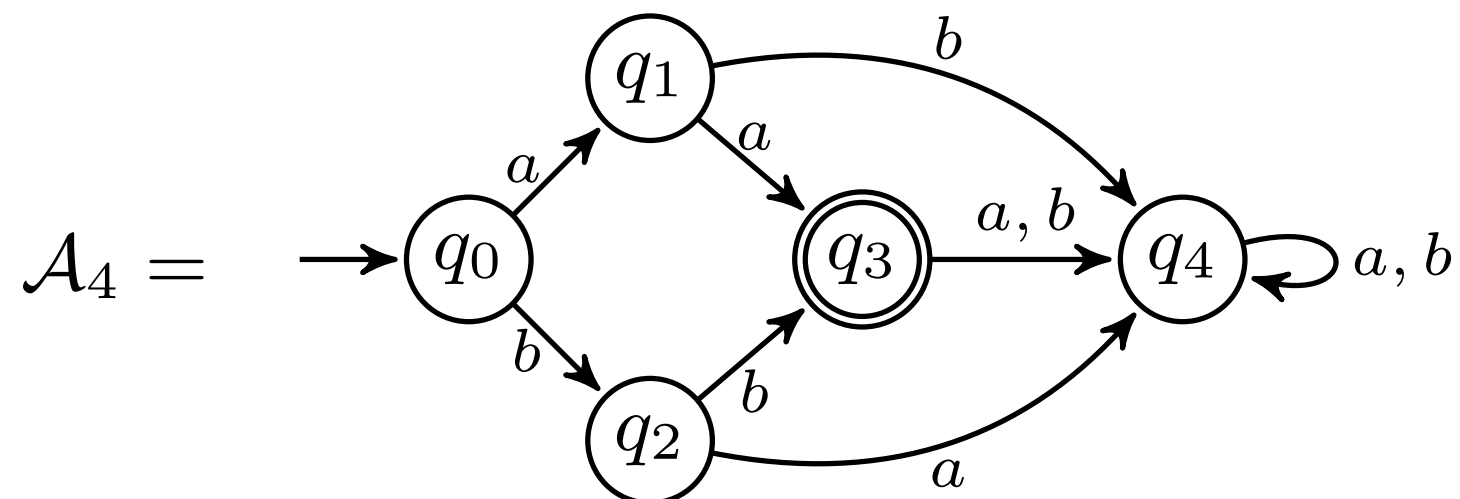
L^* , by example

| | ϵ | a |
|------------|------------|-----|
| ϵ | 0 | 0 |
| a | 0 | 1 |
| aa | 1 | 0 |
| b | 0 | 0 |
| bb | 1 | 0 |
| ab | 0 | 0 |
| aaa | 0 | 0 |
| aab | 0 | 0 |
| ba | 0 | 0 |
| bba | 0 | 0 |
| bbb | 0 | 0 |

| | ϵ | a | b |
|------------|------------|-----|-----|
| ϵ | 0 | 0 | 0 |
| a | 0 | 1 | 0 |
| aa | 1 | 0 | 0 |
| b | 0 | 0 | 1 |
| bb | 1 | 0 | 0 |
| ab | 0 | 0 | 0 |
| aaa | 0 | 0 | 0 |
| aab | 0 | 0 | 0 |
| ba | 0 | 0 | 0 |
| bba | 0 | 0 | 0 |
| bbb | 0 | 0 | 0 |



$t = babb$



Automata learning

simple is
beautiful.

&

POWERFUL

Applications : Security, formal verification, ...

Generalisations : Mealy machines, I/O automata, ...

Automata learning

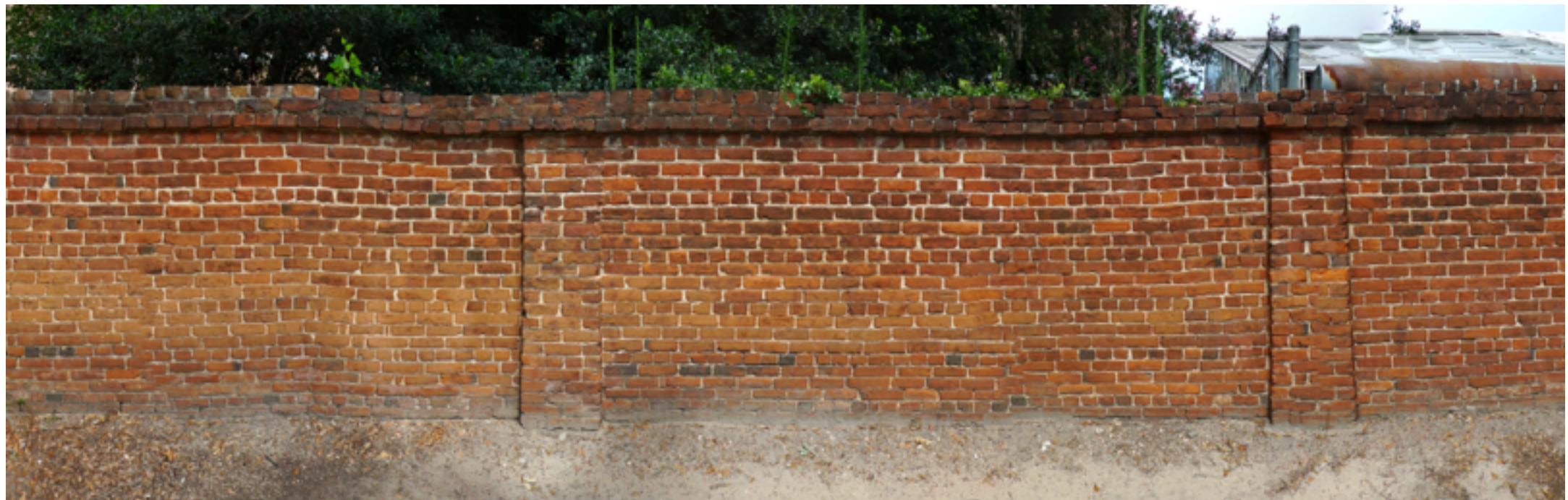
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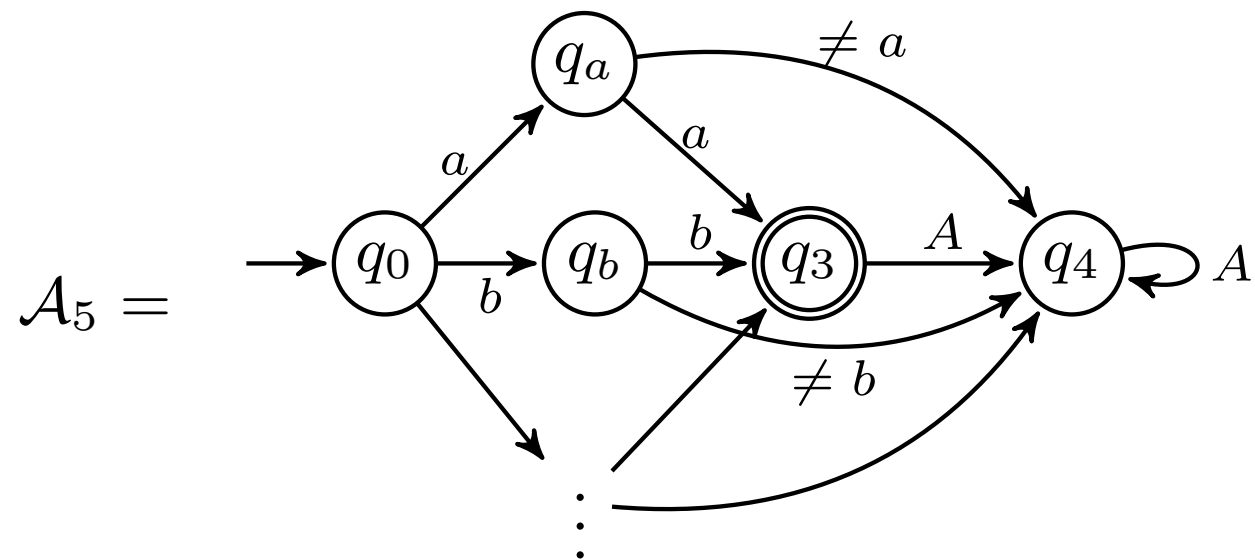


Finite alphabets are not enough in many applications

Infinite alphabets

$$\mathcal{L}_n = \{ww \mid w \in A^*, |w| = n\}$$

$$\mathcal{L}_1 = \{\bar{a}a, \bar{b}b\}$$



infinite automaton

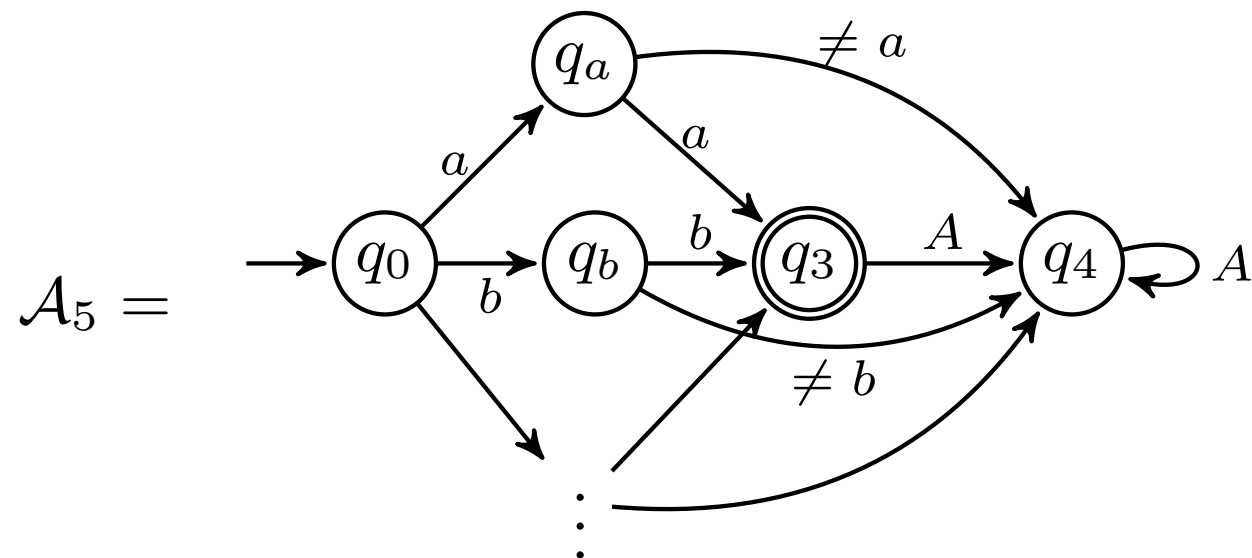
Infinite alphabets

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A infinite

$$\mathcal{L}_1 = \{\bar{a}a, \bar{b}b\}$$

$$\mathcal{L}_1 = \{aa, bb, cc, dd, \dots\}$$



infinite automaton

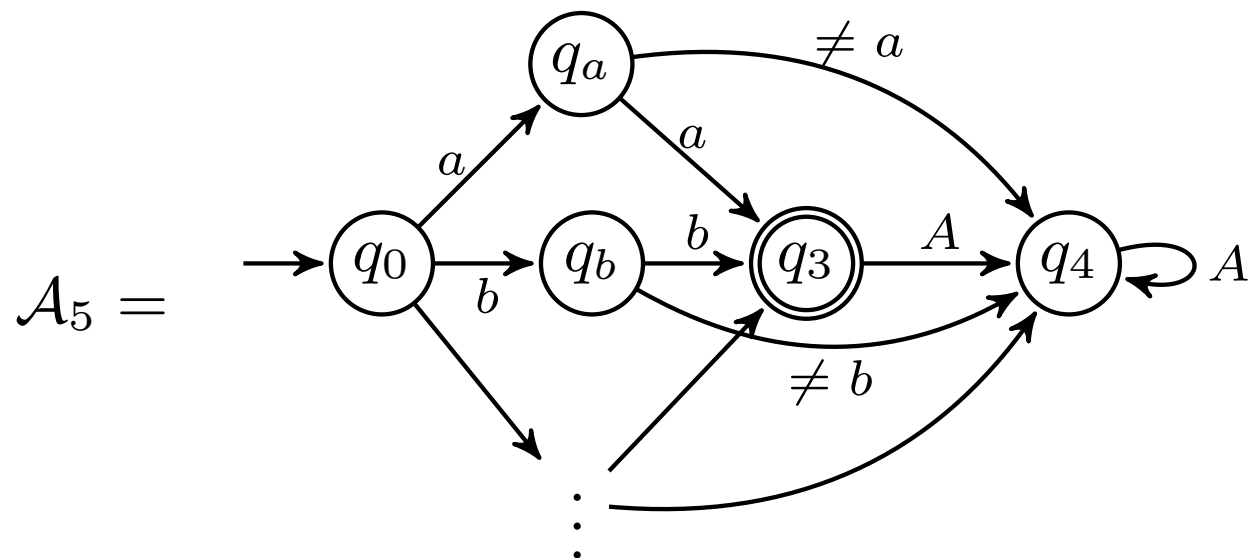
Infinite alphabets

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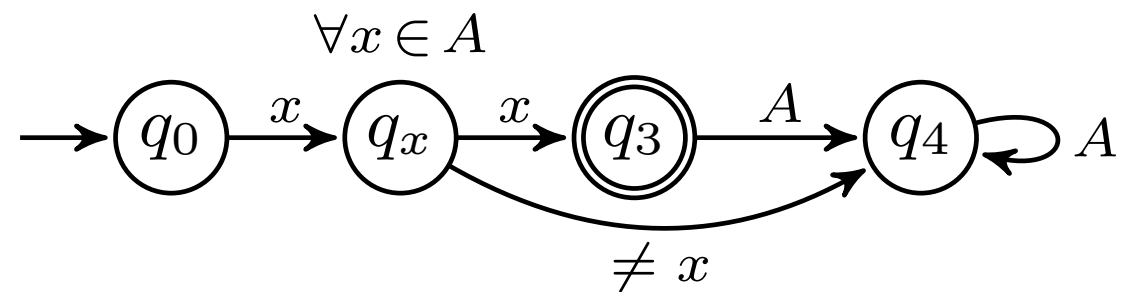
A infinite

$$\mathcal{L}_1 = \{\bar{a}a, \bar{b}b\}$$

$$\mathcal{L}_1 = \{aa, bb, cc, dd, \dots\}$$



infinite automaton



but with a finite representation

Nominal automata

Nominal sets



name binding
alpha-equivalence

.....

Nominal automata

Nominal sets



name binding
alpha-equivalence

.....

Infinite sets

Nominal automata

Nominal sets



name binding
alpha-equivalence

.....

Infinite sets with symmetries

Nominal automata

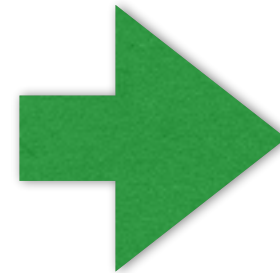
Nominal sets



name binding
alpha-equivalence

.....

Infinite sets with symmetries



Finitely representable

Nominal automata

Nominal sets



name binding
alpha-equivalence

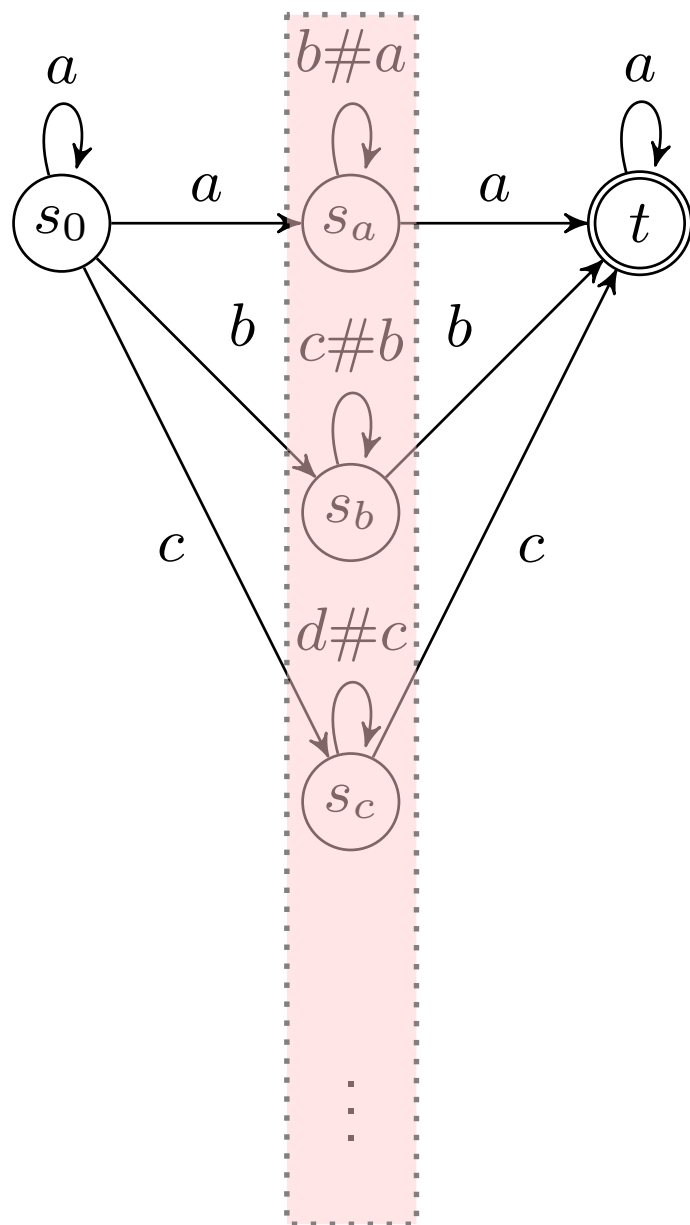
.....



Automata theory
over nominal sets

Nominal automata

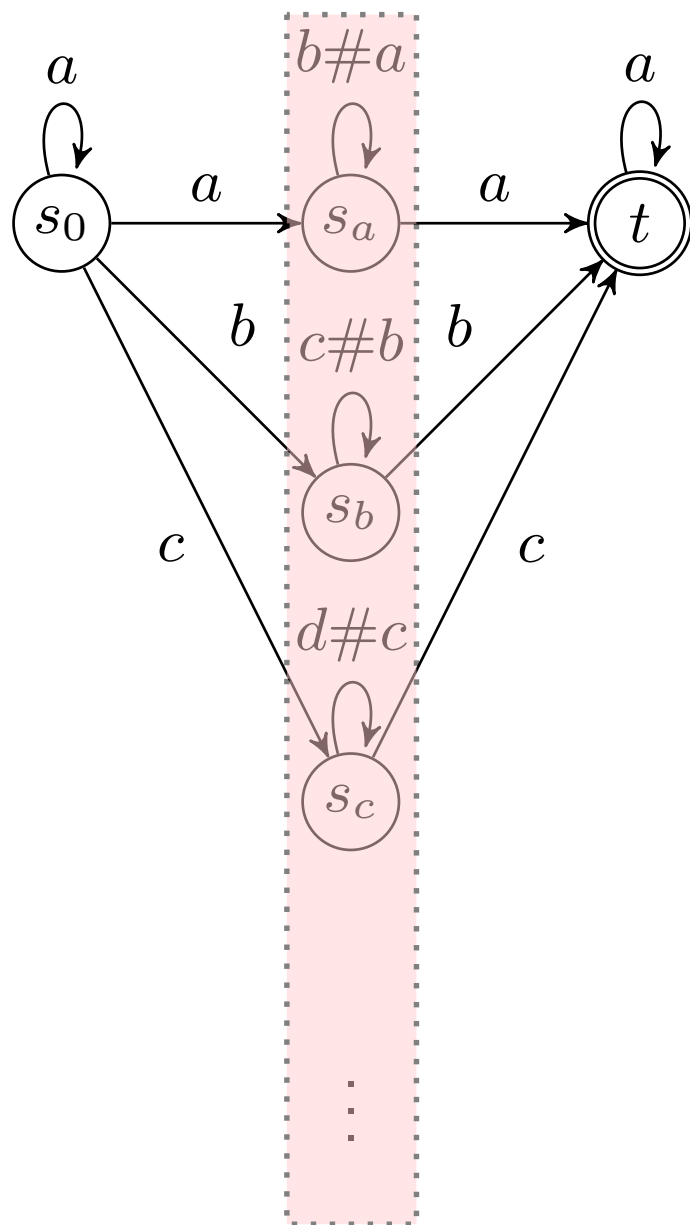
\mathbb{A} infinite



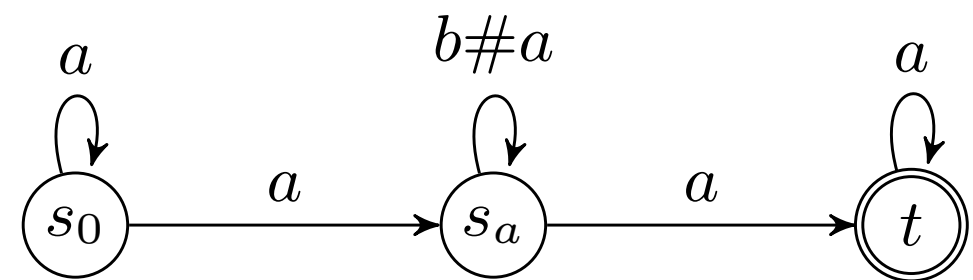
$$\{w \in \mathbb{A}^* \mid \exists a. a \text{ occurs twice in } w\}$$

Nominal automata

\mathbb{A} infinite

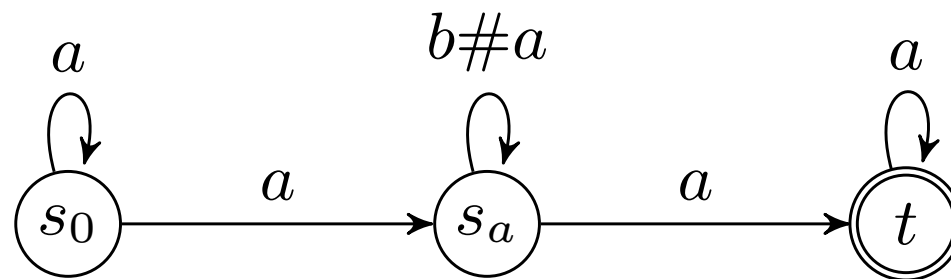


$$\{w \in \mathbb{A}^* \mid \exists a. a \text{ occurs twice in } w\}$$



finite representation

Nominal automata



finite representation

$$X = \{s_0\} + \mathbb{A} + \{t\}$$

canonical permutations

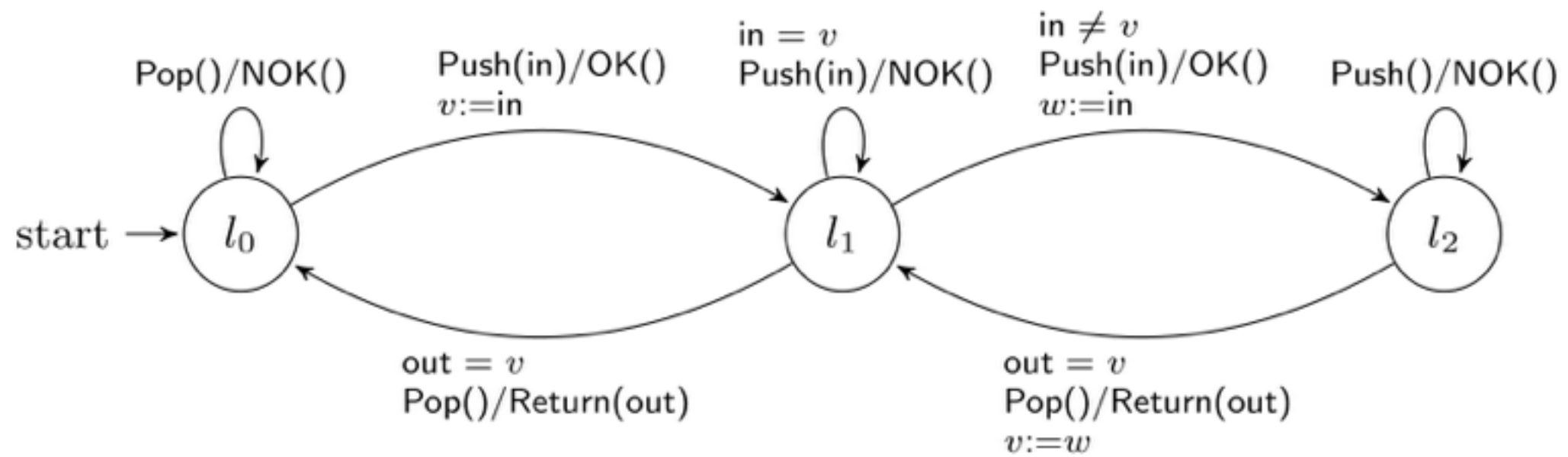
$$\pi : \mathbb{A} \rightarrow \mathbb{A}$$

$$s_a \mapsto s_{\pi a}$$

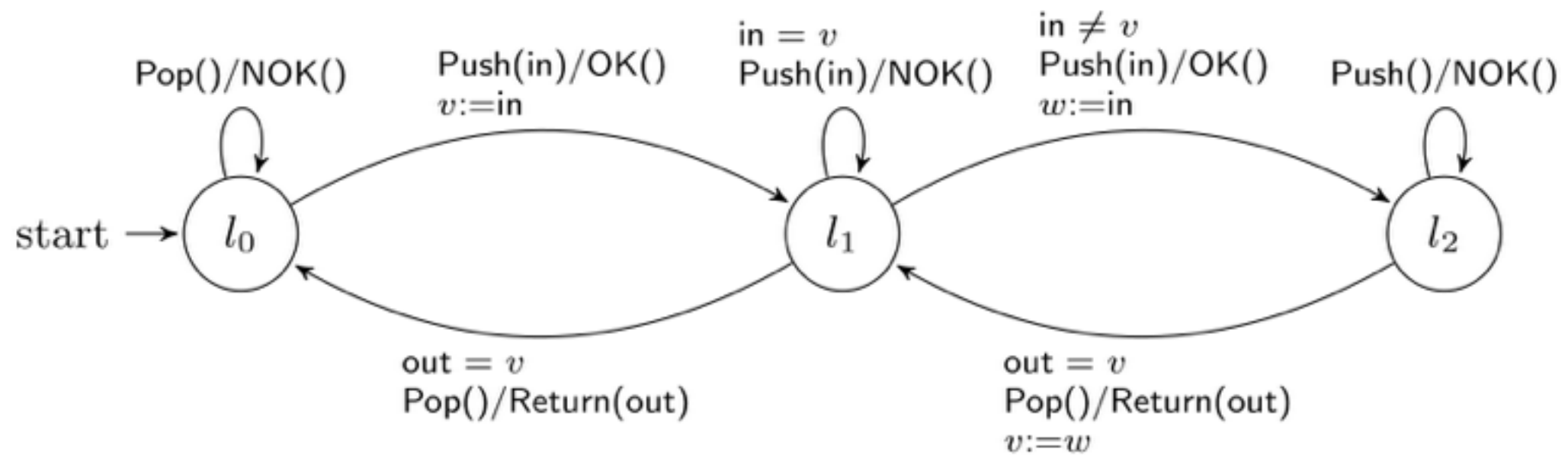
transition closed under permutations
equivariant

$$s_a \xrightarrow{a} t \Rightarrow s_{\pi a} \xrightarrow{\pi a} t$$

Nominal automata

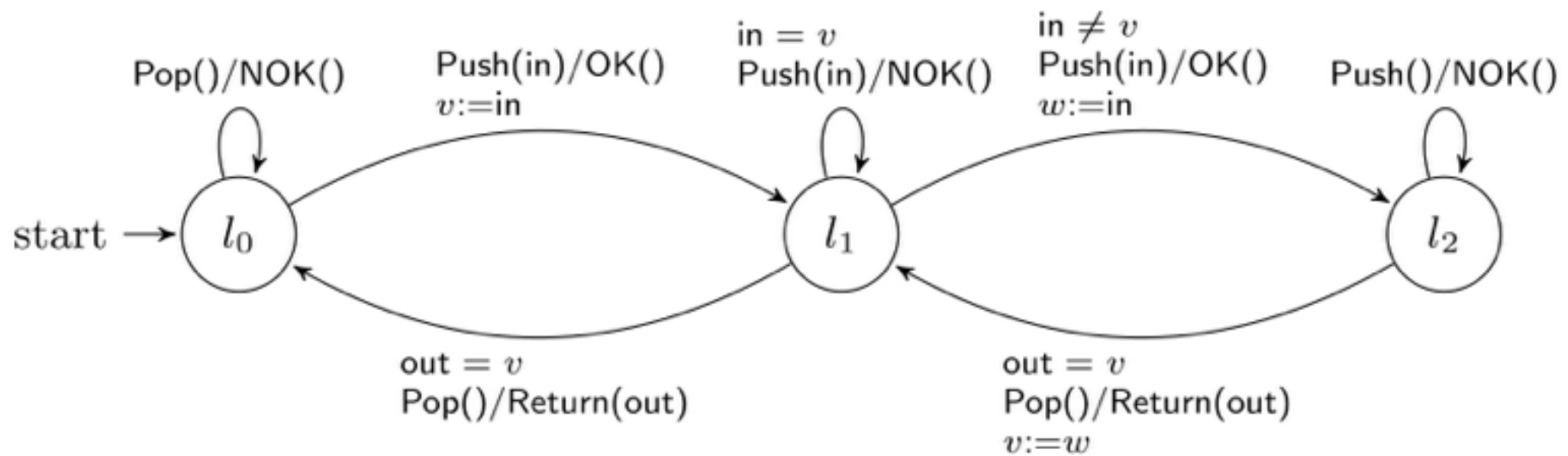


Nominal automata



Better or worse than Nominal automata?

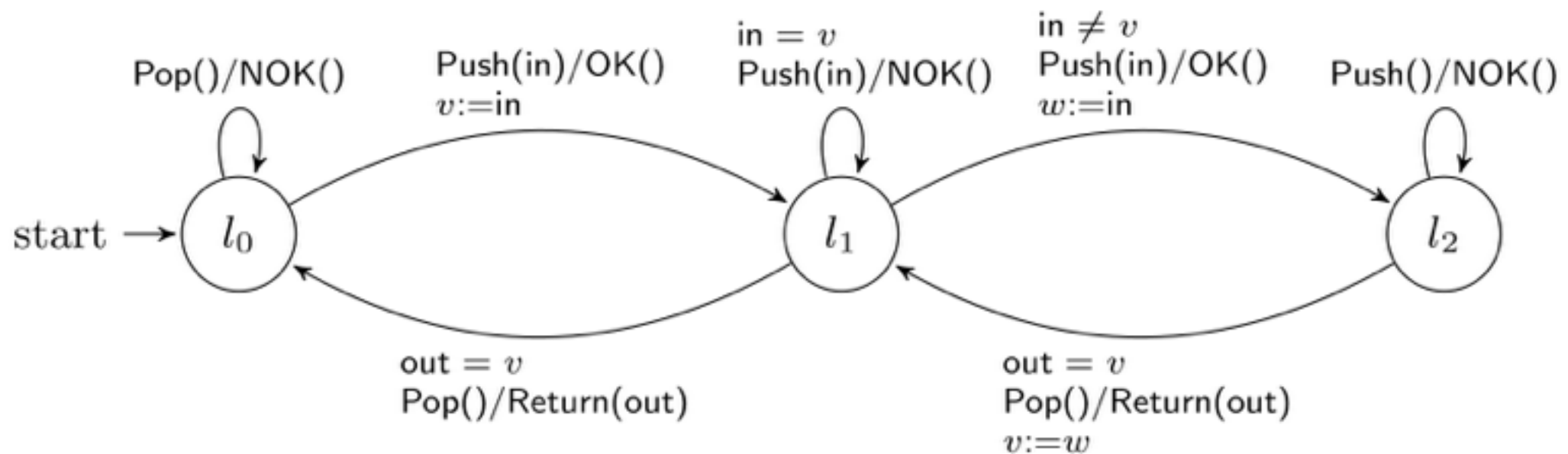
Nominal automata



Better or worse than Nominal automata?

Nominal automata are *just* automata in **Nom**

Nominal automata



Better or worse than Nominal automata?

Nominal automata are *just* automata in **Nom**

$\lambda x.\mathbf{Nom}(x)$ research program

This talk

Learning algorithm for nominal automata

This talk

Learning algorithm for nominal automata

using category theory & coalgebra

This talk

Learning algorithm for nominal automata

using category theory & coalgebra

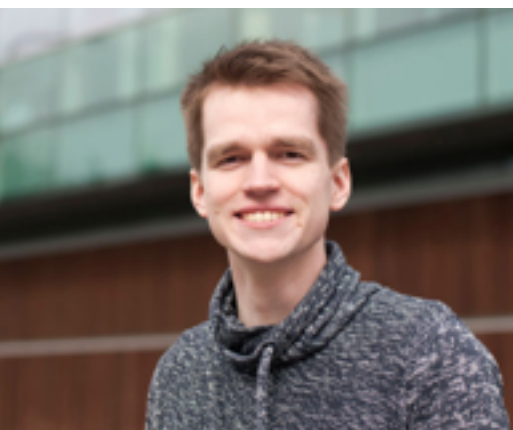
Angluin meets the Warsaw group

$$(\lambda x. \mathbf{Nom}(x))(L^*)$$

Credits

Joshua Moerman, Matteo Sammartino, Alexandra Silva, Bartek Klin, Michal Szynwelski. **Learning Nominal Automata.**

Bart Jacobs, Alexandra Silva. **Automata Learning: A Categorical Perspective.**



Challenges

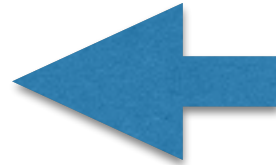
L^* LEARNER

```
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```

Challenges

L* LEARNER

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10   Make the conjecture  $M(S, E)$ 
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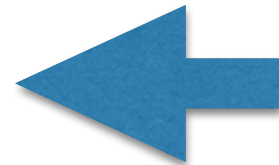


range over infinite sets

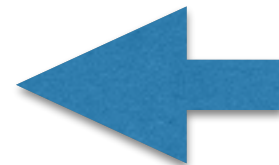
Challenges

L* LEARNER

```
1   $S, E \leftarrow \{\epsilon\}$ 
2  repeat
3    while  $(S, E)$  is not closed or not consistent
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5      find  $s_1 \in S, a \in A$  such that
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range over infinite sets



finding witnesses potentially
requires checking infinite rows

Challenges

L* LEARNER

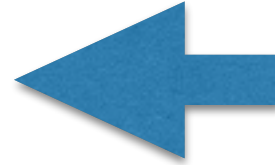
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range over infinite sets



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t has only finitely many prefixes,
but an infinite S is necessary

Challenges

L* LEARNER

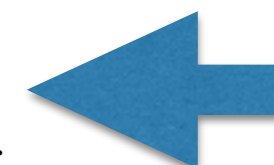
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range over infinite sets



finding witnesses potentially
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no finite automaton accepts \mathcal{L}_1

Challenges

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range over infinite sets

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(P1) the sets S , $S \cdot A$ and E admit a finite representation up to permutations;

(P2) row is such that $row(\pi(s))(\pi(e)) = row(s)(e)$, for all $s \in S$ and $e \in E$.
Observation table admits a finite symbolic representation.

Nominal L^*

$$6' \quad S \leftarrow S \cup \text{orb}(sa)$$

$$9' \quad E \leftarrow E \cup \text{orb}(ae)$$

$$12' \quad E \leftarrow E \cup \text{prefixes}(\text{orb}(t))$$

only 3 lines changed!

Nominal L^*

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only 3 lines changed!

not really... all definitions have to be adapted
to nominal/equivariant.

Nominal L^*

$$\begin{array}{ll} 6' & S \leftarrow S \cup \text{orb}(sa) \\ 9' & E \leftarrow E \cup \text{orb}(ae) \\ 12' & E \leftarrow E \cup \text{prefixes}(\text{orb}(t)) \end{array}$$

only 3 lines changed!

not really... all definitions have to be adapted
to nominal/equivariant.

Correctness, termination, ... have to be re-proved!

Nominal L^*



not really.

pted

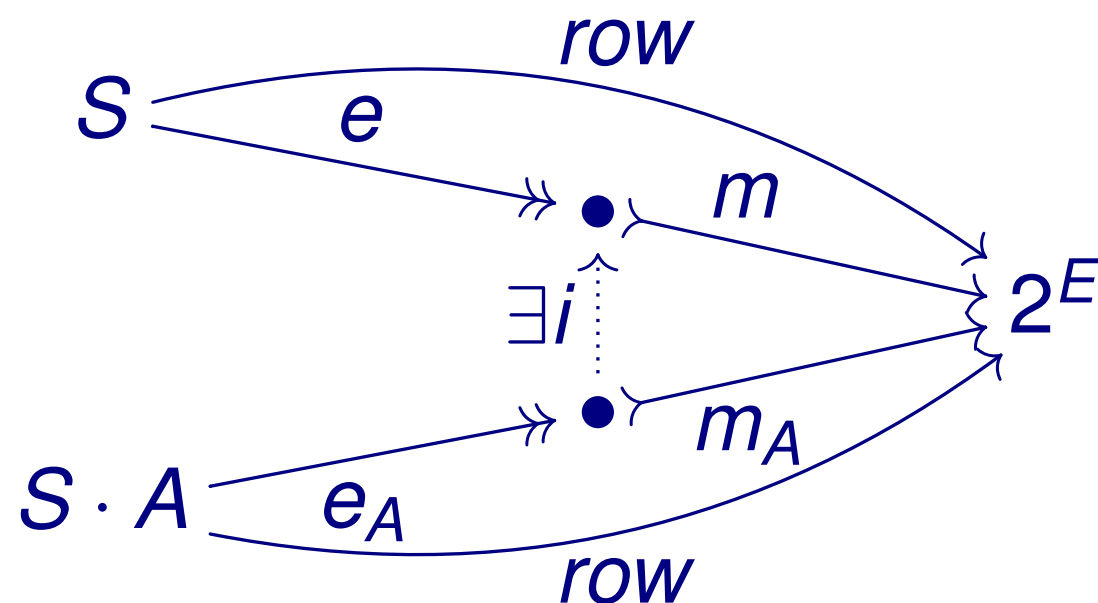
Correctness

-proved!

Categorical glasses

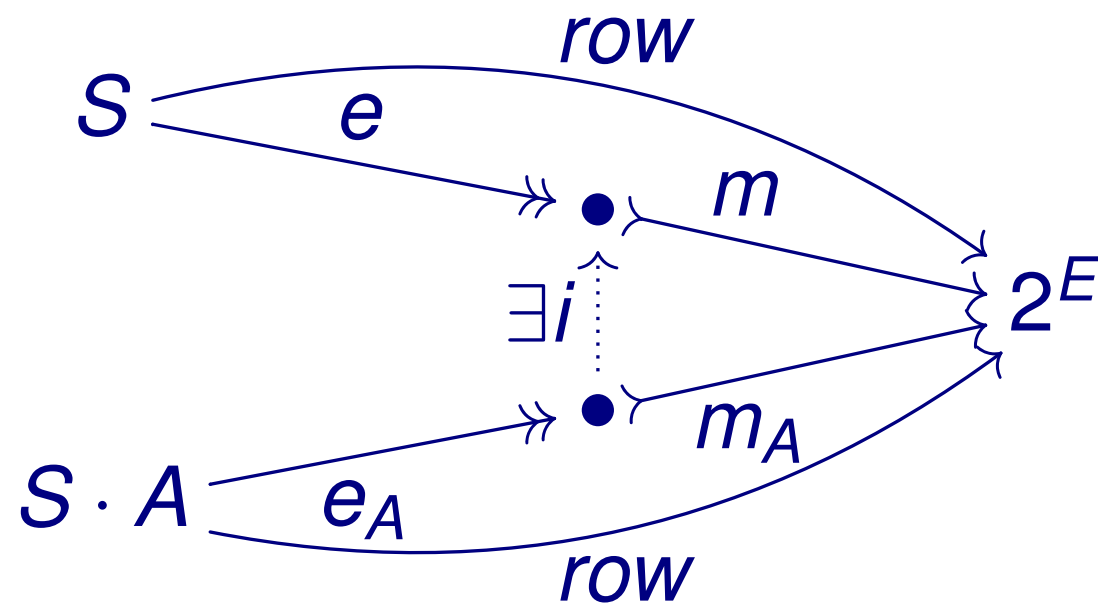
(S, E, row) is *closed* if for all $t \in S \cdot A$ there exists an $s \in S$ such that $\text{row}(t) = \text{row}(s)$.

Categorical glasses



(S, E, row) is *closed* if for all $t \in S.A$ there exists an $s \in S$ such that $\text{row}(t) = \text{row}(s)$.

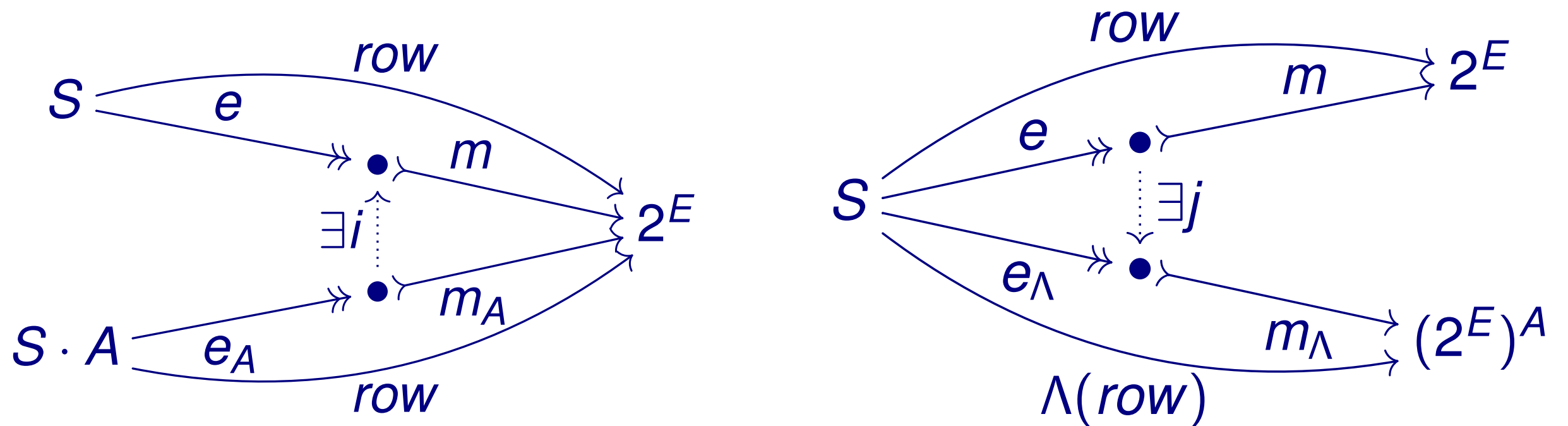
Categorical glasses



(S, E, row) is *closed* if for all $t \in S \cdot A$ there exists an $s \in S$ such that $row(t) = row(s)$.

(S, E, row) is *consistent* if whenever $s_1, s_2 \in S$ are such that $row(s_1) = row(s_2)$, for all $a \in A$, $row(s_1 a) = row(s_2 a)$.

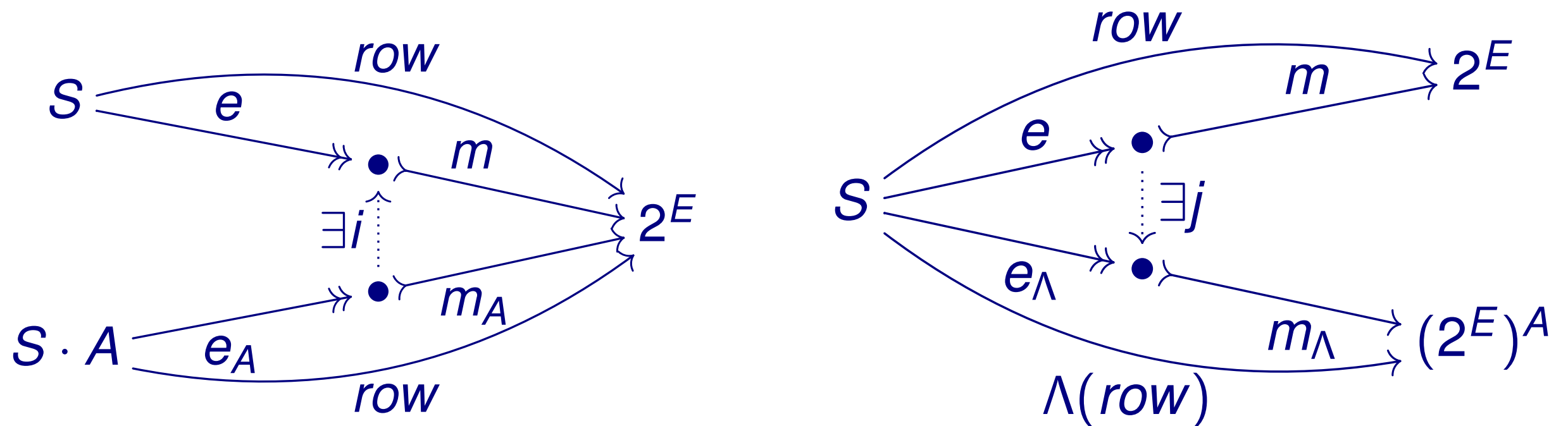
Categorical glasses



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Categorical glasses



(S, E, row) is *closed* if for all $t \in C \cdot A$ there exists an $s \in S$ such that

Pretty.... but is it useful?

(S, E, row) is *consistent* if whenever $s_1, s_2 \in S$ are such that $row(s_1) = row(s_2)$, for all $a \in A$, $row(s_1 a) = row(s_2 a)$.

The power of abstraction

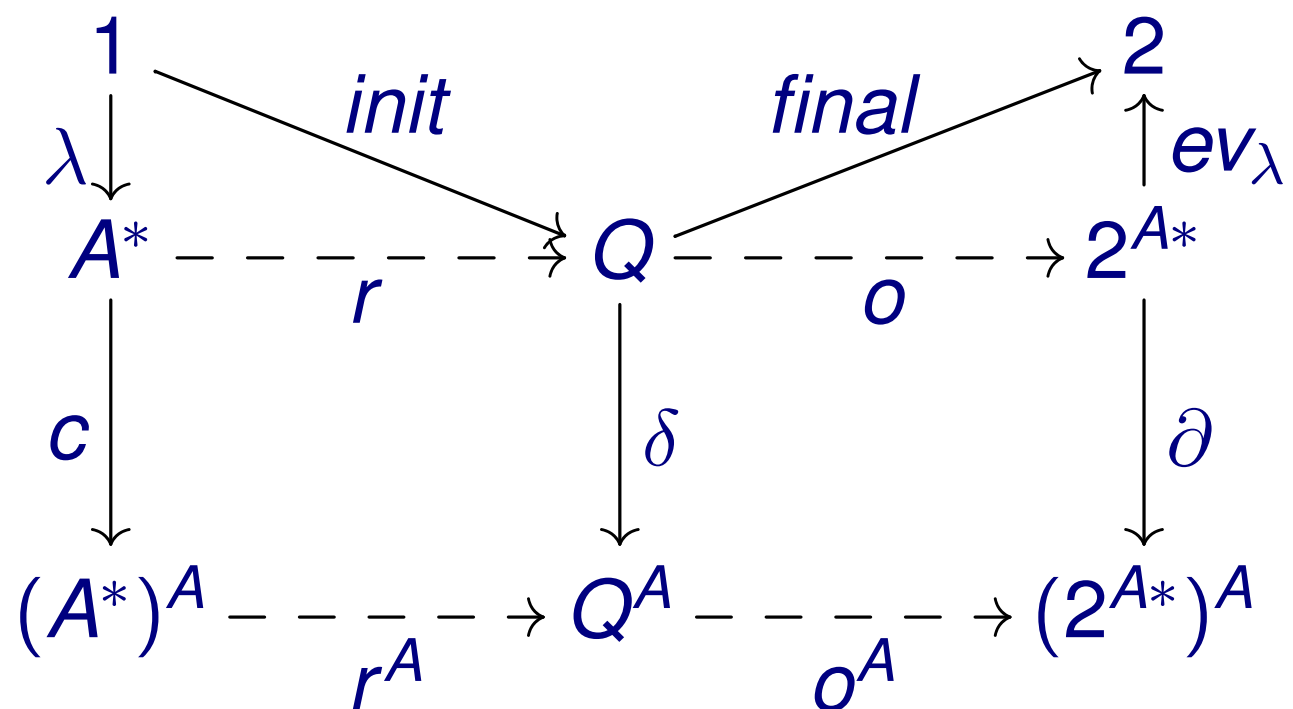
Definitions are the *same*

Proof of correctness is the *same*

The power of abstraction

Definitions are the *same*

Proof of correctness is the *same*



The power of abstraction

Learning weighted automata = vector spaces

2014, Jacobs&Silva

Learning NFAs = join semi lattices

2009, Bollig, Habermehl, Kern, Leucker - Angluin-style learning for NFA

Learning product automata = products

Implicitly done by Rivest, Schapire (diversity based learning)

All of these only work for ordinary alphabets

In Nom...

In Nom...

Correctness was easy

Brings almost nothing new to the table

Easy to implement variations

For example, different counter example analysis

Works for any symmetry $Aut(\mathcal{M})$ for any ω -categorical model \mathcal{M}

For example homogeneous structures: $(\mathbb{N}, =)$, (\mathbb{Q}, \leq) , (\mathcal{R}, adj) , ...

Needed for products to be orbit-finite

In Nom...

In Nom...

Implementation was not easy

(Partly because the library for nominal computation was young)

Implementation is not efficient

No concrete communication with teacher yet

(But theoretically possible)

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Abstraction is guidance but there is no free lunch!

Future Work

Implementation

Succinctness

Other symmetries

Other tools

Verification

Conclusions

simple is
beautiful.

&

POWERFUL