# Coalgebra at the CWI: a brief overview

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Centrum Wiskunde & Informatica, The Netherlands

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# The team

Jan Rutten

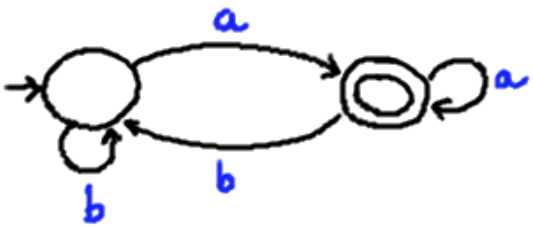
Marcello Bonsangue

Alexandra Silva

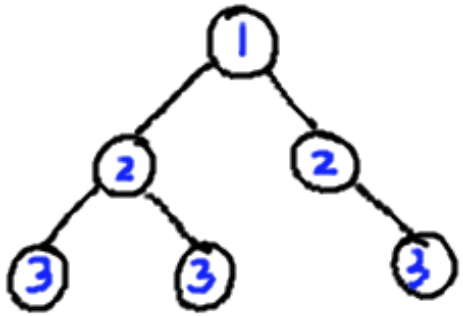
Joost Winter

# What is Coalgebra?

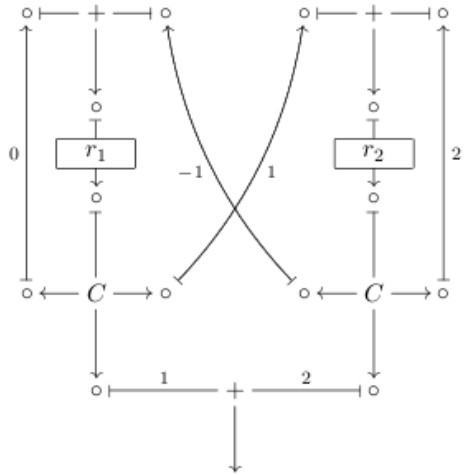
- Mathematical framework to reason about several dynamical systems and models of computation




det. automata




trees




linear systems/
stream circuits

# How do we do it?

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$$(S, g\colon S \to G(S))$$

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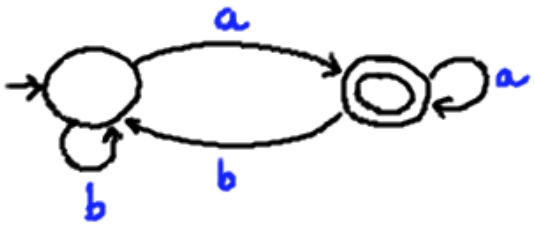


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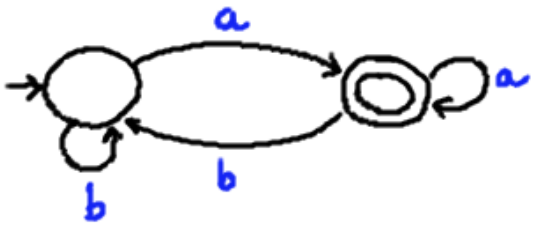


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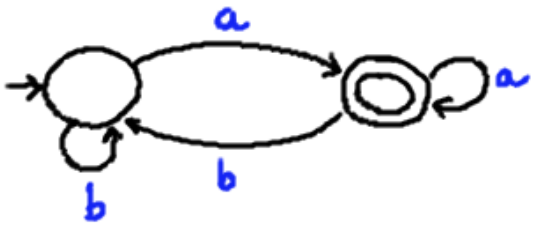


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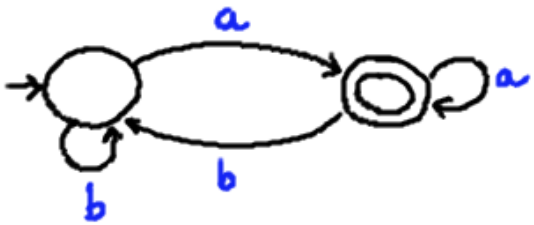


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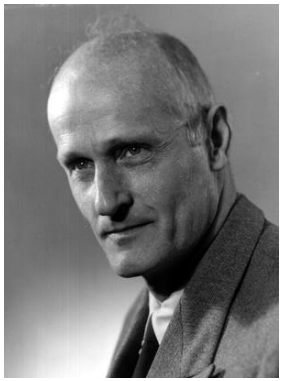
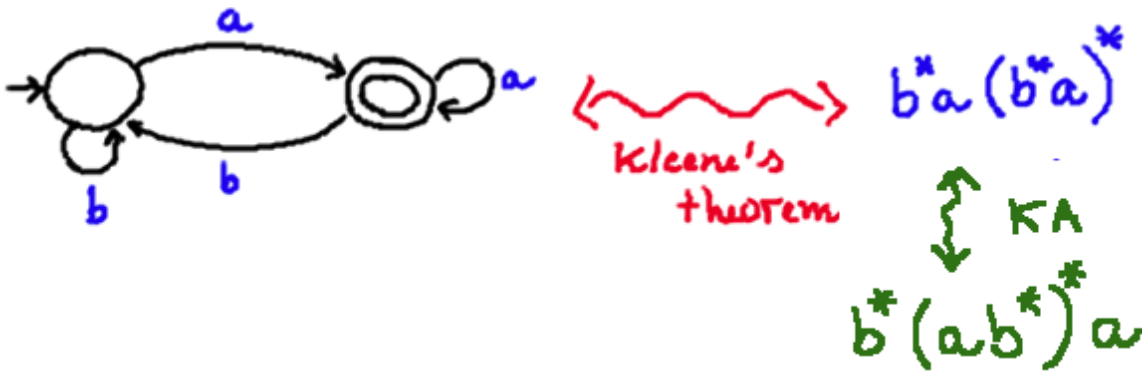
**Example:** Languages ($A^*$) and language equivalence

# Some of our recent achievements



S. Kleene




D. Kozen

## Bonsangue, Rutten & Silva 2009

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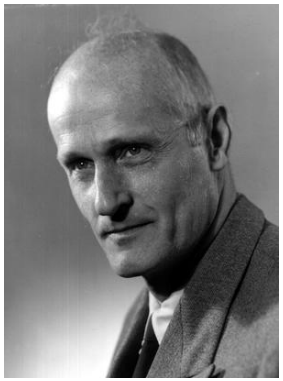

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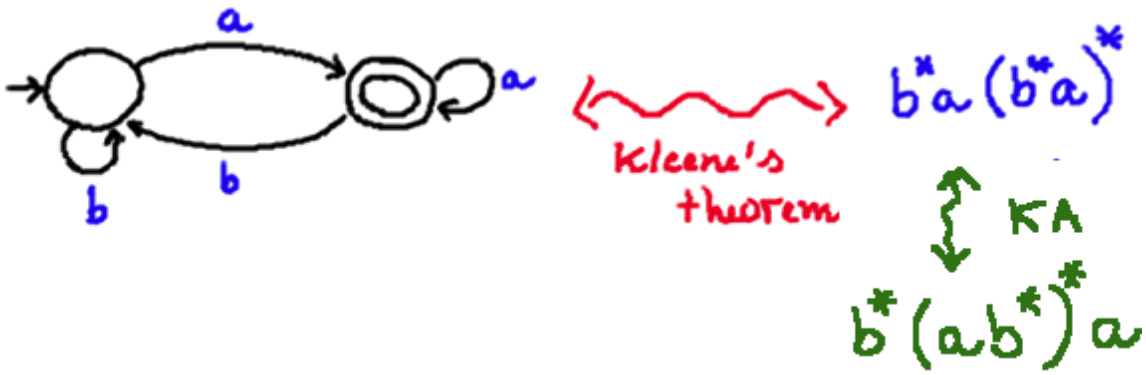


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# What does this yield concretely?

- Languages, axiomatizations and algorithms to reason about equivalence for a large class of models
- From the general framework we recover known results (e.g. for LTS Milner's language and axiomatization), but also. . .
- . . . new results, impact in the concurrency community (cf. our CONCUR paper–Bonchi, Bonsangue, Rutten & Silva 2009)

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# What we are looking at now

## Context Free languages and grammars

- Is it possible to generalize CFL to other models?

## Automation

- Coinduction is very suitable for automation
- Can we provide automatic reasoning on equivalence of models?
- Applications in program schematology, compiler certification, etc

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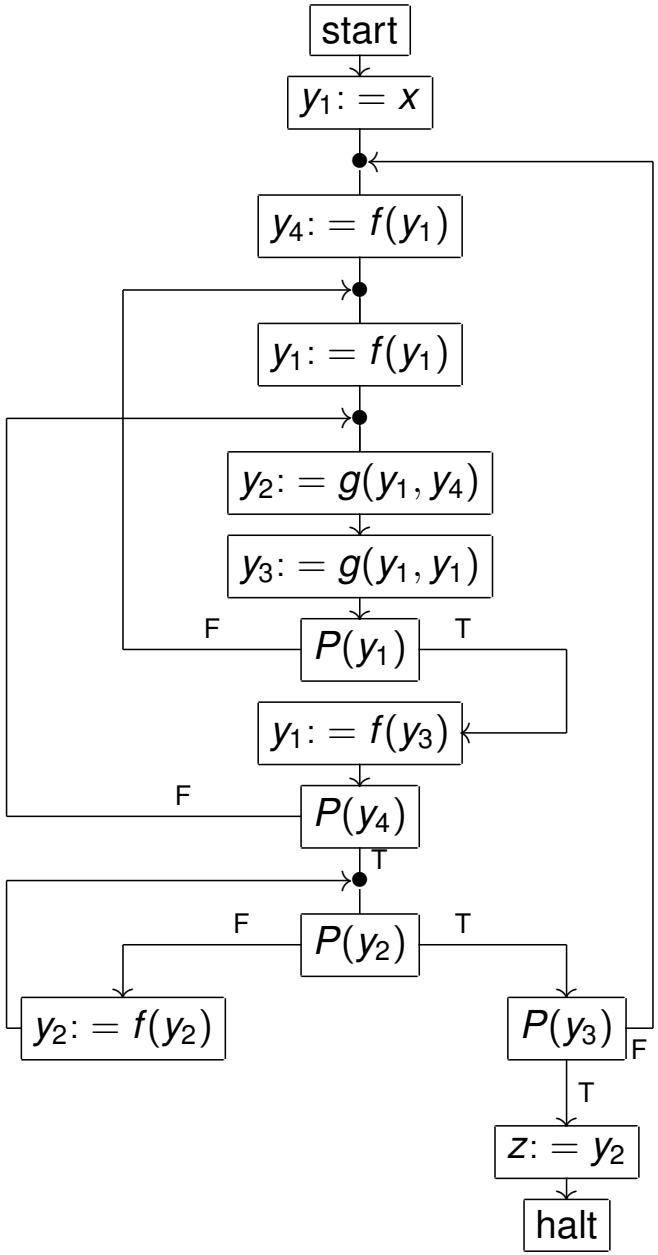


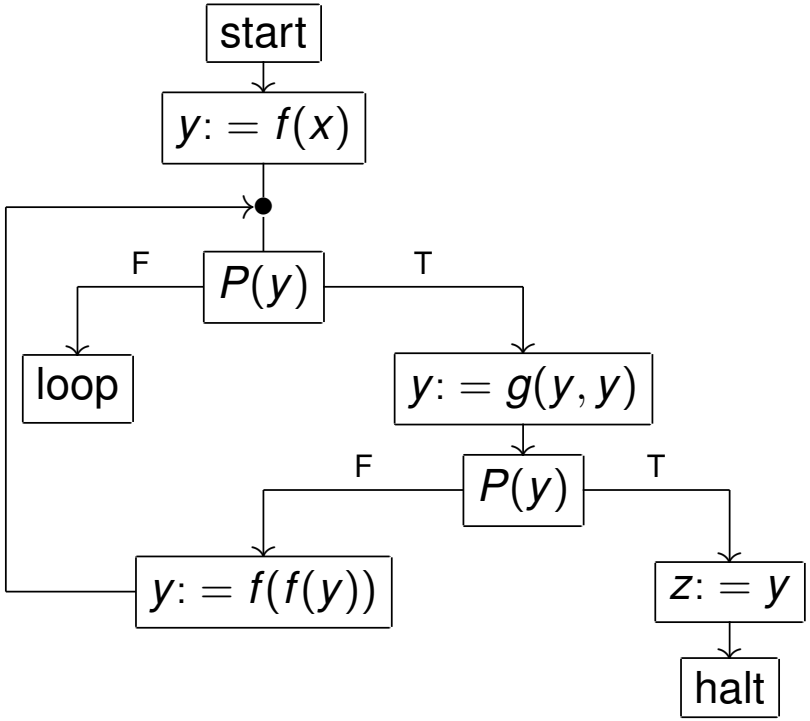


Algebraic proof long and requires ingenuity

Coinductive proof fully automatic

# Thank you for your attention!