Coalgebraic Logics and Synthesis for Mealy Machines

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FoSSaCS 2008

Budapest, April 2008

Motivation

Design of Sequential systems

- (Binary) Mealy machines $\leftrightarrow$ digital circuits
- There is no formal way of specifying Mealy machines
- Typically they are "defined" in a natural language, such as English

  Source of ambiguities
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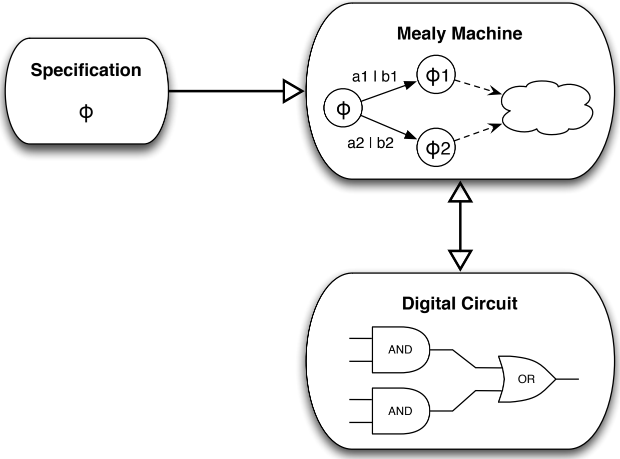

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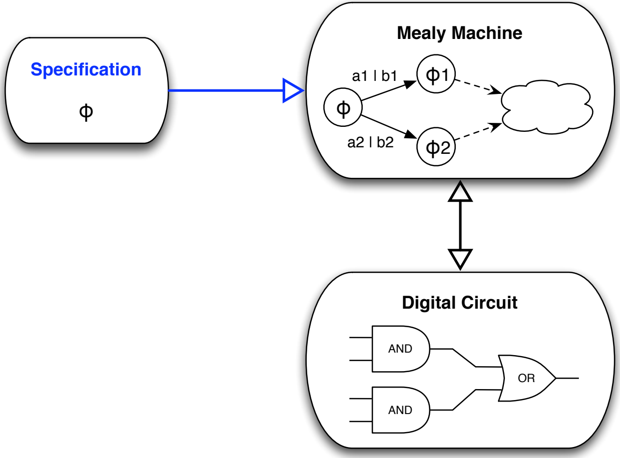

But first...

What do we mean by **Binary Mealy Machine?**

- Mealy machines = **Deterministic Mealy machines**
- Mealy machine = set of states S + transition function f

$$\begin{aligned} f &: S \to (B \times S)^A \\ f(s)(a) &= <b, s'> \end{aligned}$$

$$s \xrightarrow{a|b} s'$$

A is the input alphabet and B is the output alphabet

- Binary Mealy machines : $A = 2$ and

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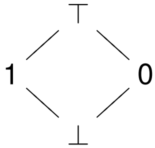
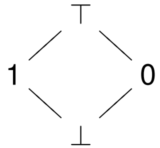


Information order

$\mathcal{B} =$



- $\top$ – *abstraction* (under-specification)
- $\bot$ – *inconsistency* (over-specification)
- 0 and 1 – concrete output values.

Mealy automata are coalgebras

Observation:

A Mealy machine is a coalgebra of the functor

$$\begin{aligned} M &: Set \to Set \\ M(X) &= (B \times X)^A \end{aligned}$$

(Almost) for free:

- Notion of (bi)simulation : equivalence between states, minimization
- Semantics in terms of final coalgebra – causal functions
- Specification language for Mealy machines

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And . . . What can we do now?

Simple properties

- Output 0 at each input of 1

$$1 \downarrow 0$$

- Output 0 at each input of two consecutive 1's

$$\nu x.(1(1 \downarrow 0 \wedge 1(x) \wedge 0(x)) \wedge 0(x))$$

- Output 0 at each second input of 1

$$\nu x.(0(x) \wedge 1(\nu y.0(y) \wedge 1 \downarrow 0 \wedge 1(x))$$

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$$\lambda \ : \ L \to (B \times L)^A$$

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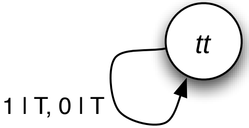

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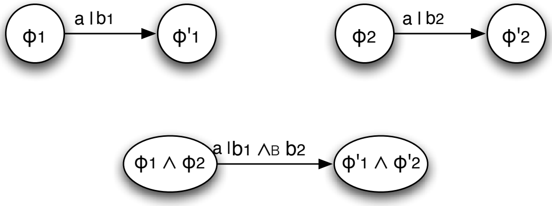

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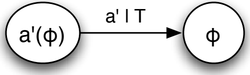

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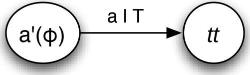

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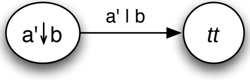

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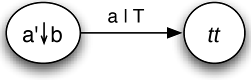

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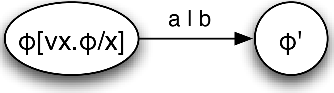

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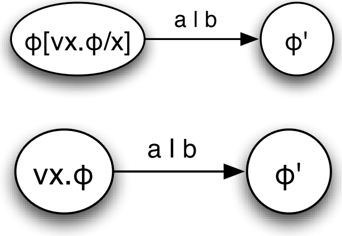

Why a coalgebra structure?

Two advantages

1. Semantics

$$\begin{array}{ccc} L & \xrightarrow{[\![\cdot]\!]} & \Gamma \\ \lambda \downarrow & & \downarrow \gamma \\ (B \times L)^A & \longrightarrow & (B \times \Gamma)^A \end{array}$$

2. Satisfaction relation in terms of simulation

$$s \models \phi \Leftrightarrow s \lesssim \phi$$

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Logic is expressive

Theorem

1. *For all states s, s' of a Mealy machine (S, f),*

$$s \sim s' \text{ iff } \forall_{\phi \in L}.s \models \phi \Leftrightarrow s' \models \phi.$$

2. *If S is finite then there exists for any $s \in S$ a formula ϕ_s such that $s \sim \phi$.*

We also want synthesis:

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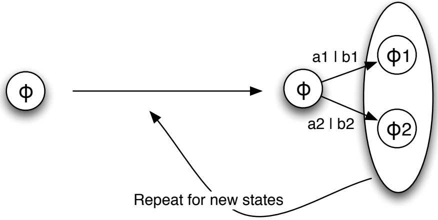
Easy answer: Apply λ repeatedly!



- λ will not deliver a finite automaton.

$$\phi = \nu x.1(x \wedge (\nu y.1(y)))$$

$$\lambda(\phi)(1) = < \top, \phi \wedge (\nu y.1(y)) >$$
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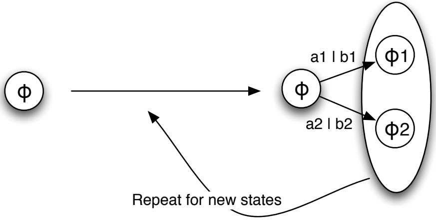

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Normalization

$$\begin{array}{lcl}
norm(tt) & = & tt \\
norm(a(\phi)) & = & a(norm(\phi)) \\
norm(a \downarrow b) & = & a \downarrow b \\
norm(\phi_1 \wedge \phi_2) & = & conj(rem(flatten(norm(\phi_1) \wedge norm(\phi_2)))) \\
norm(\nu x.\phi) & = & \nu x.norm(\phi)
\end{array}$$

$$norm(a(tt) \wedge a \downarrow b \wedge tt \wedge a \downarrow b) = norm(a(tt) \wedge a \downarrow b)$$

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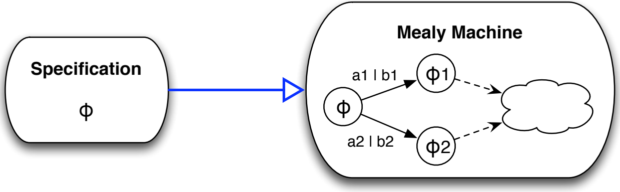

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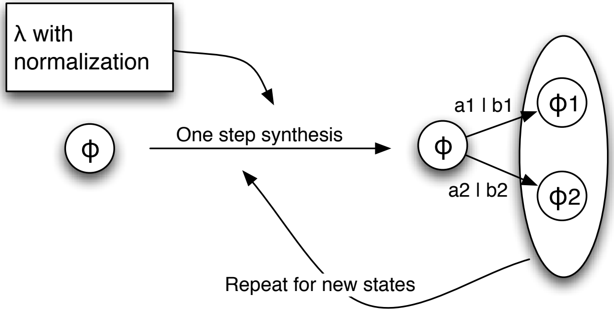
λ with
normalization
ϕ
One step synthesis
ϕ
a1 | b1
a2 | b2
ϕ1
ϕ2
Repeat for new states

One-step synthesis

$$
\begin{array}{lcl}
\delta & : & L \rightarrow (B \times L)^A \\
\delta(tt) & = & <\top, tt> \\
\delta(a'(\phi))(a) & = & \begin{cases} <\top, norm(\phi)> & a = a' \\ <\top, tt> & otherwise \end{cases} \\
\delta(a' \downarrow b)(a) & = & \begin{cases} <b, tt> & a = a' \\ <\top, tt> & otherwise \end{cases} \\
\delta(\phi_1 \wedge \phi_2)(a) & = & \delta(\phi_1)(a) \sqcap \delta(\phi_2)(a) \\
\delta(\nu x.\phi)(a) & = & <b, norm(\phi')> \\
 & & where <b, \phi'>= \delta(\phi[\nu x.\phi/x])(a)
\end{array}
$$

$$
<b_1, \phi_1> \quad \sqcap \quad <b_2, \phi_2> = <b_1 \wedge_B b_2, norm(\phi_1 \wedge \phi_2)>
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Examples

- $\phi = 1 \downarrow 0 \wedge (\nu x.1(x))$

$$\begin{aligned}
\delta(\phi)(0) &= \;<\top, tt> \\
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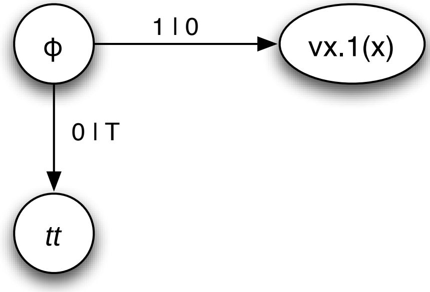

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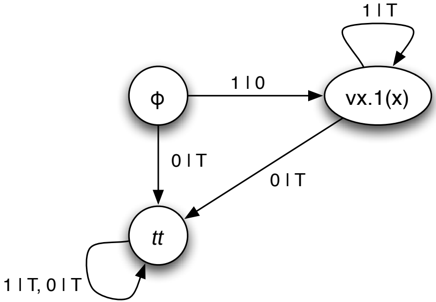

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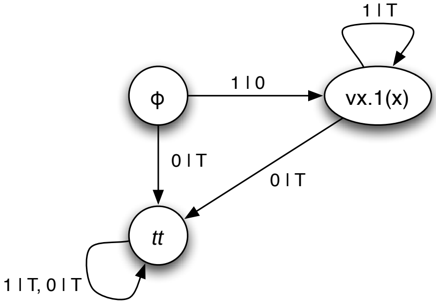

Remark: Not minimal.

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- $\phi_2 = \nu x.(1(1 \downarrow 0) \wedge 1(x))$

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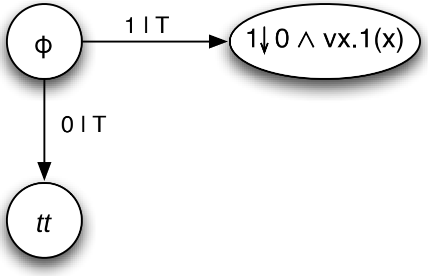
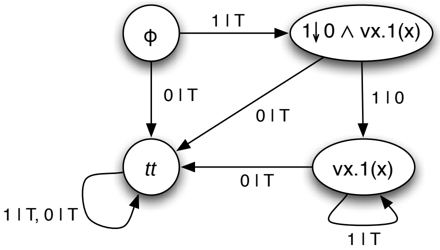


Examples

- $\phi_2 = \nu x.(1(1 \downarrow 0) \wedge 1(x))$

$$\begin{aligned} \delta(\phi_2)(0) &= <\top, tt> \\ \delta(\phi_2)(1) &= \delta(1(1 \downarrow 0))(1) \sqcap \delta(\nu x.1(x))(1) \\ &= <\top, 1 \downarrow 0> \sqcap <\top, \nu x.1(x)> \\ &= <\top, 1 \downarrow 0 \wedge (\nu x.1(x))> \end{aligned}$$



Examples

- $\phi_3 = \nu x.1(x \wedge (\nu y.1(y) \wedge 1 \downarrow 0))$

$$
\begin{aligned}
\delta(\phi_3)(0) &= <\top, tt> \\
\delta(\phi_3)(1) &= <\top, \phi_3 \wedge (\nu y.1(y) \wedge 1 \downarrow 0)>
\end{aligned}
$$

Examples

- $\phi_3 = \nu x.1(x \wedge (\nu y.1(y) \wedge 1 \downarrow 0))$

$$\begin{array}{rcl} \delta(\phi_3)(0) & = & < \top, tt > \\ \delta(\phi_3)(1) & = & < \top, \phi_3 \wedge (\nu y.1(y) \wedge 1 \downarrow 0) > \end{array}$$

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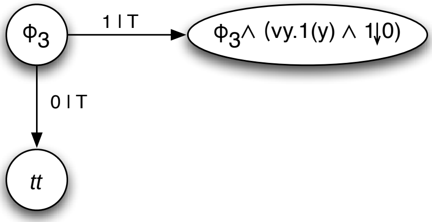
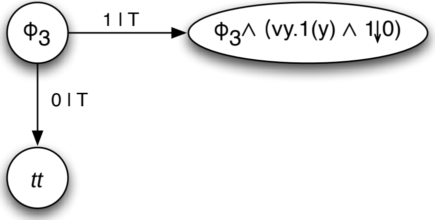


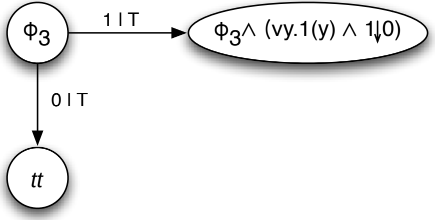
Examples (cont.)

$$\delta(\phi_3 \wedge (\nu y.1(y) \wedge 1 \downarrow 0))(1)$$



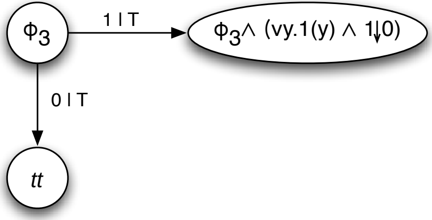
Examples (cont.)

$$\begin{aligned} & \delta(\phi_3 \wedge (\nu y.1(y) \wedge 1 \downarrow 0))(1) \\ = \; & \delta(\phi_3)(1) \sqcap \delta(\nu y.1(y) \wedge 1 \downarrow 0)(1) \end{aligned}$$



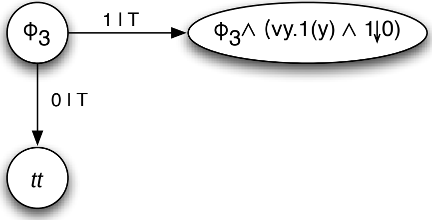
Examples (cont.)

$$\begin{aligned} & \delta(\phi_3 \wedge (\nu y.1(y) \wedge 1 \downarrow 0))(1) \\ = \; & \delta(\phi_3)(1) \sqcap \delta(\nu y.1(y) \wedge 1 \downarrow 0)(1) \\ = \; & < \top, \phi \wedge (\nu y.1(y) \wedge 1 \downarrow 0) > \sqcap < 0, \nu y.1(y) \wedge 1 \downarrow 0 > \end{aligned}$$



Examples (cont.)

$$\begin{array}{ll} & \delta(\phi_3 \wedge (\nu y.1(y) \wedge 1 \downarrow 0))(1) \\ = & \delta(\phi_3)(1) \sqcap \delta(\nu y.1(y) \wedge 1 \downarrow 0)(1) \\ = & < \top, \phi \wedge (\nu y.1(y) \wedge 1 \downarrow 0) > \sqcap < 0, \nu y.1(y) \wedge 1 \downarrow 0 > \\ = & < 0, norm(\phi_3 \wedge (\nu y.1(y) \wedge 1 \downarrow 0) \wedge (\nu y.1(y) \wedge 1 \downarrow 0)) > \end{array}$$



Examples (cont.)

$$
\begin{aligned}
& \delta(\phi_3 \wedge (\nu y.1(y) \wedge 1 \downarrow 0))(1) \\
= \; & \delta(\phi_3)(1) \sqcap \delta(\nu y.1(y) \wedge 1 \downarrow 0)(1) \\
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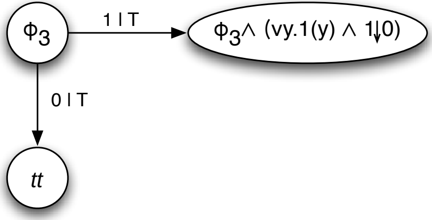
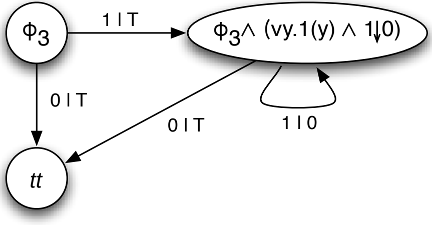
= \; & < 0, \phi_3 \wedge (\nu y.1(y) \wedge 1 \downarrow 0) >
\end{aligned}
$$



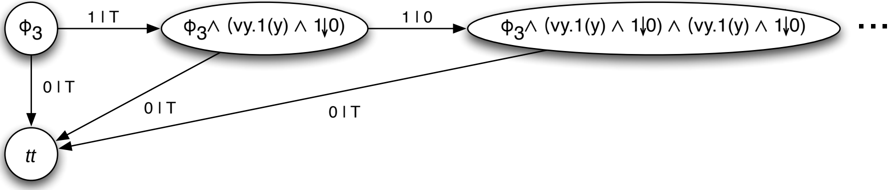
Examples (cont.)

$$
\begin{array}{rl}
& \delta(\phi_3 \wedge (\nu y.1(y) \wedge 1 \downarrow 0))(1) \\
= & \delta(\phi_3)(1) \sqcap \delta(\nu y.1(y) \wedge 1 \downarrow 0)(1) \\
= & < \top, \phi \wedge (\nu y.1(y) \wedge 1 \downarrow 0) > \sqcap < 0, \nu y.1(y) \wedge 1 \downarrow 0 > \\
= & < 0, norm(\phi_3 \wedge (\nu y.1(y) \wedge 1 \downarrow 0) \wedge (\nu y.1(y) \wedge 1 \downarrow 0)) > \\
= & < 0, \phi_3 \wedge (\nu y.1(y) \wedge 1 \downarrow 0) >
\end{array}
$$



Examples (cont.)

$$\begin{aligned} & \delta(\phi_3 \wedge (\nu y.1(y) \wedge 1 \downarrow 0))(1) \\ = \;& \delta(\phi_3)(1) \sqcap \delta(\nu y.1(y) \wedge 1 \downarrow 0)(1) \\ = \;& < \top, \phi \wedge (\nu y.1(y) \wedge 1 \downarrow 0) > \sqcap < 0, \nu y.1(y) \wedge 1 \downarrow 0 > \\ = \;& < 0, \mathit{norm}(\phi_3 \wedge (\nu y.1(y) \wedge 1 \downarrow 0) \wedge (\nu y.1(y) \wedge 1 \downarrow 0)) > \\ = \;& < 0, \phi_3 \wedge (\nu y.1(y) \wedge 1 \downarrow 0) > \end{aligned}$$



Conclusions

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- Coalgebraic approach : bisimulation and logics
- New logic for Mealy machines
- Synthesis algorithm that produces a finite machine

Future Work

- The logic has exactly the operators needed to represent all Mealy machines
- Similar to regular expressions for deterministic automata
- Can we generalize this approach to other types of coalgebras?

Towards Regular expressions for polynomial coalgebras

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