Algebra-Coalgebra Duality: applications in automata theory

Alexandra Silva

Radboud Universiteit Nijmegen and CWI

VU TCS seminar, 10 January 2014

The global message

- Two views on many problems: Algebra and coalgebra.
- The combination is essential!
- Coalgebra is semantics but also algorithms.

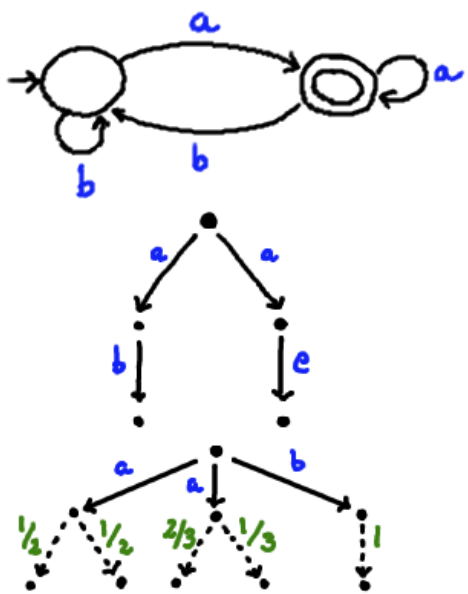
(Co)algebra

Specify and reason about systems.

(Co)algebra

Specify and reason about systems.

state-machines
e.g. DFA, LTS, PA



(Co)algebra

Specify and reason about systems.

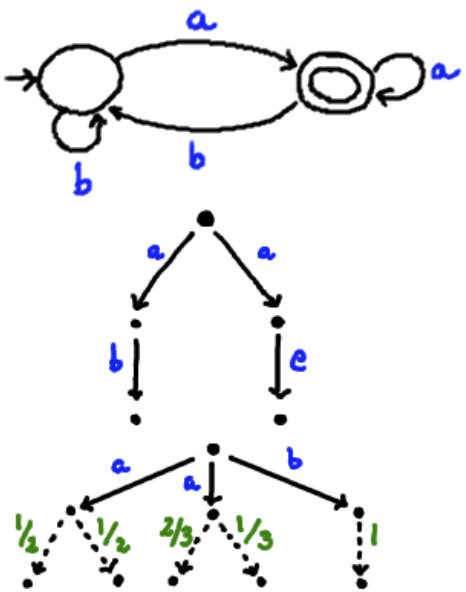
Syntax
RE, CCS, . . .

$b^* a (b^* a)^*$

$a.b.0 + a.c.0$

$a.(\frac{1}{2}.0 \oplus \frac{1}{2}.0) + \cdots$

state-machines
e.g. DFA, LTS, PA



(Co)algebra

Specify and **reason** about **systems**.

Syntax
RE, CCS, ...

$b^* a (b^* a)^*$

$a.b.0 + a.c.0$

$a.(\frac{1}{2}.0 \oplus \frac{1}{2}.0) + \cdots$

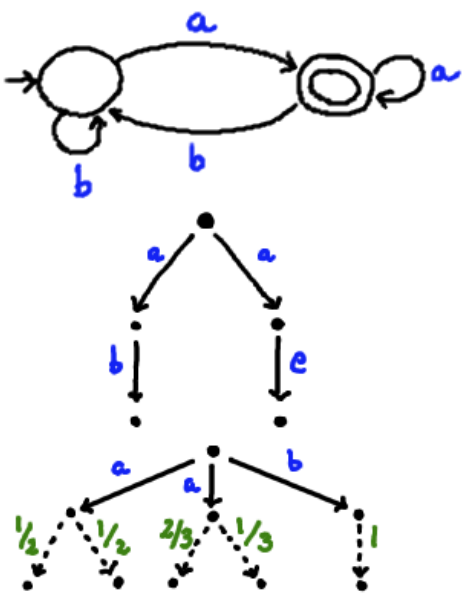
Axiomatization
KA,...

$1 + a\,a^* = a^*$
$\vdots$

$P + 0 = P$
$\vdots$

$p.P \oplus p'.P = (p + p').P$
$\vdots$

state-machines
e.g. DFA, LTS, PA



(Co)algebra

Specify and **reason** about **systems**.

Syntax
RE, CCS, ...

$b^* a (b^* a)^*$

$a.b.0 + a.c.0$

$a.(\frac{1}{2}.0 \oplus \frac{1}{2}.0) + \cdots$

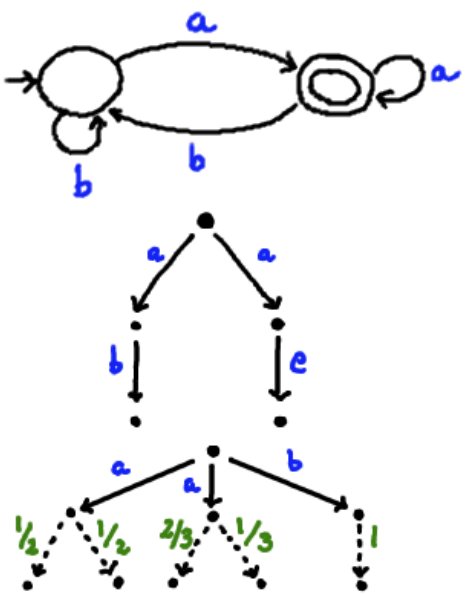
Axiomatization
KA,...

$1 + a\,a^* = a^*$
$\vdots$

$P + 0 = P$
$\vdots$

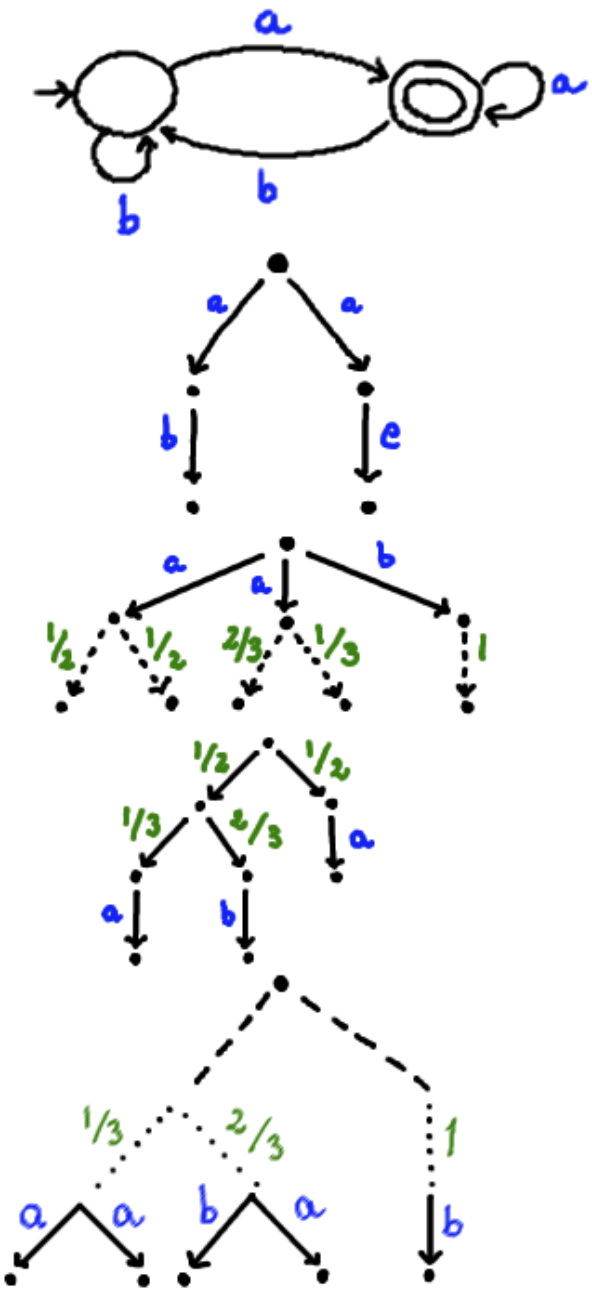
$p.P \oplus p'.P = (p + p').P$
$\vdots$

state-machines
e.g. DFA, LTS, PA



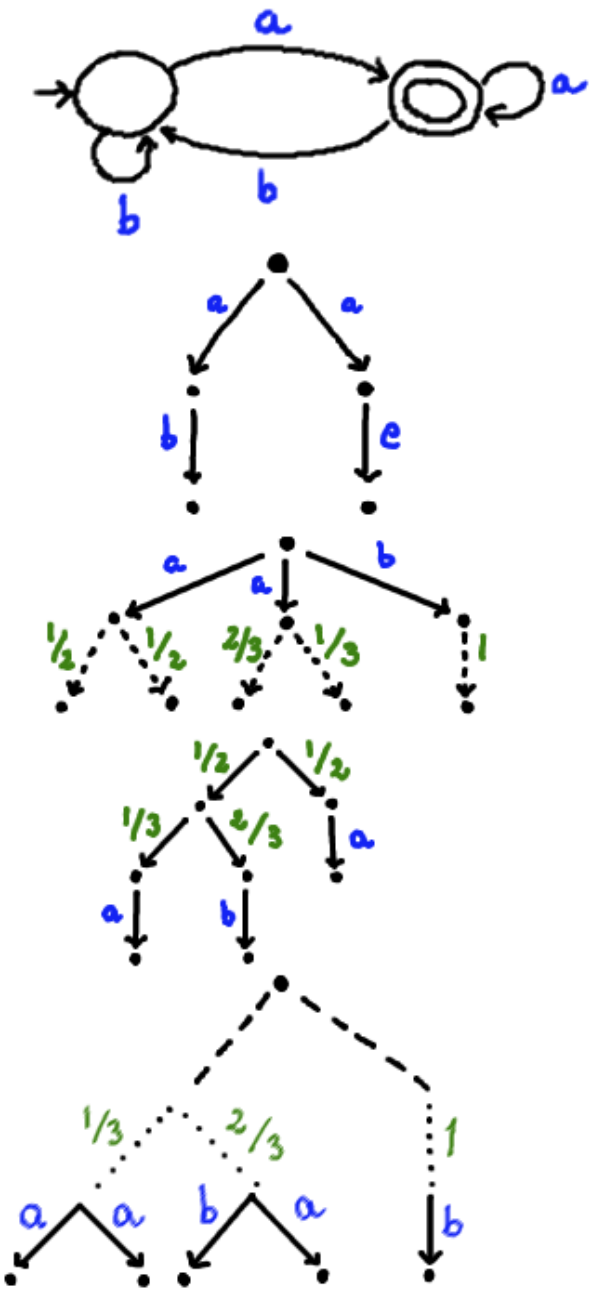
Can we do all of this uniformly in a single framework?

What do this things have in common?



$$(S, t : S \to 2 \times S^A)$$

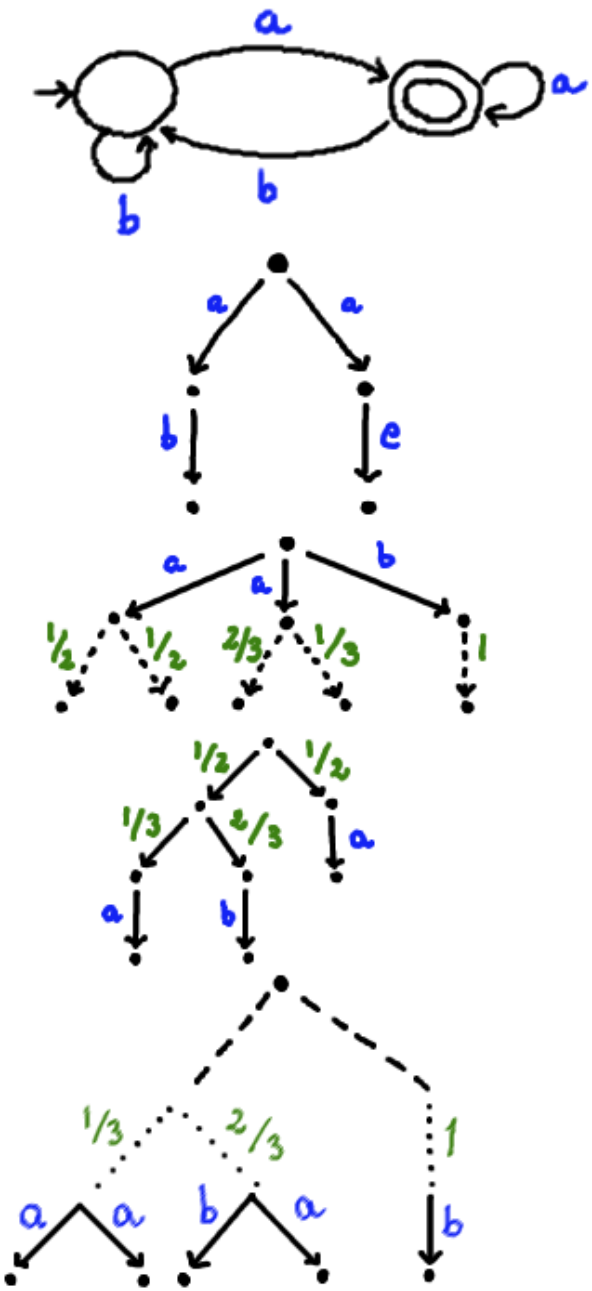
What do this things have in common?



$(S, t : S \to 2 \times S^A)$

$(S, t : S \to \mathcal{P}S^A)$

What do this things have in common?

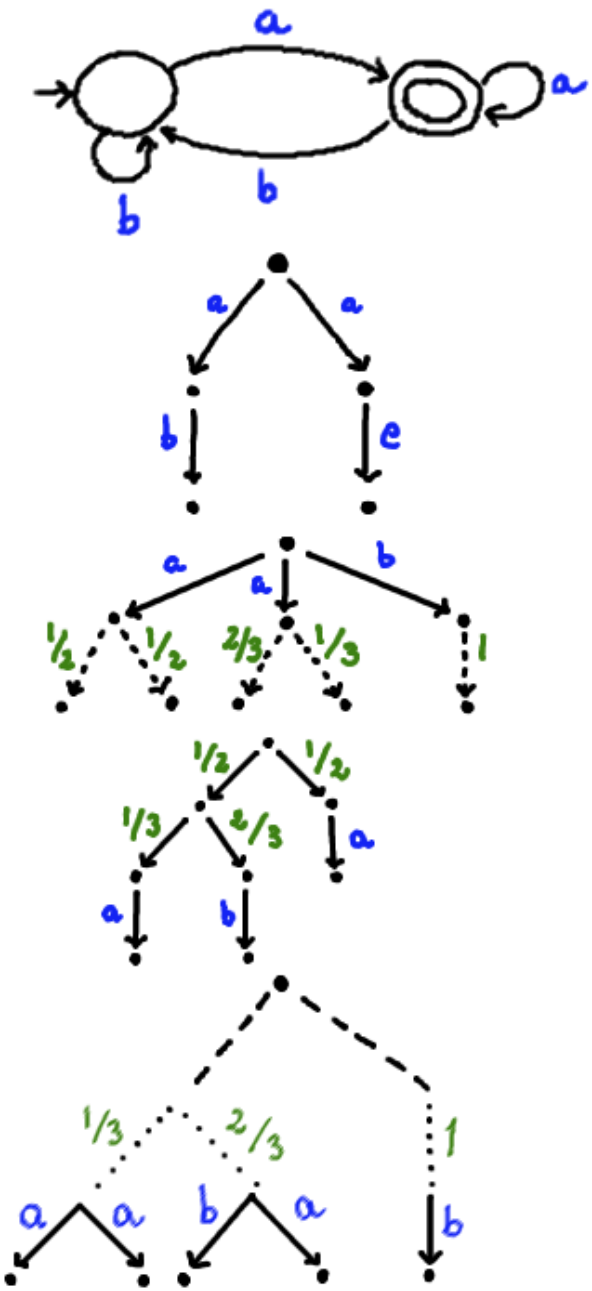


$(S, t : S \to 2 \times S^A)$

$(S, t : S \to \mathcal{P}S^A)$

$(S, t : S \to \mathcal{P}\mathcal{D}_\omega(S)^A)$

What do this things have in common?



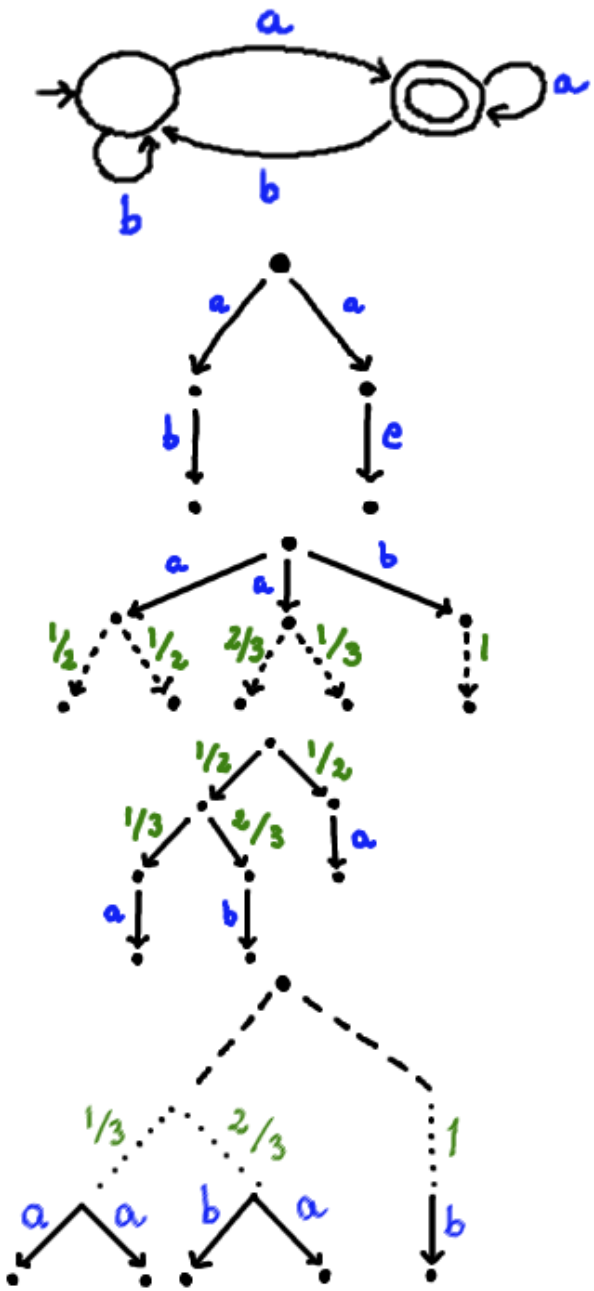
$(S, t : S \to 2 \times S^A)$

$(S, t : S \to \mathcal{P}S^A)$

$(S, t : S \to \mathcal{P}\mathcal{D}_\omega(S)^A)$

$(S, t : S \to \mathcal{D}_\omega(S) + (A \times S) + 1)$

What do this things have in common?



$(S, t : S \to 2 \times S^A)$

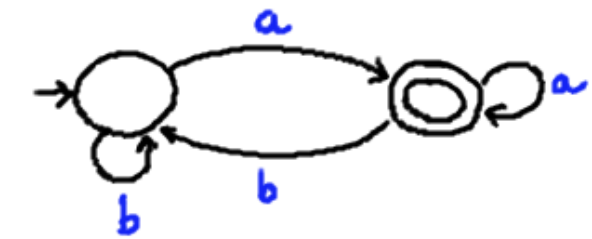
$(S, t : S \to \mathcal{P}S^A)$

$(S, t : S \to \mathcal{P}\mathcal{D}_\omega(S)^A)$

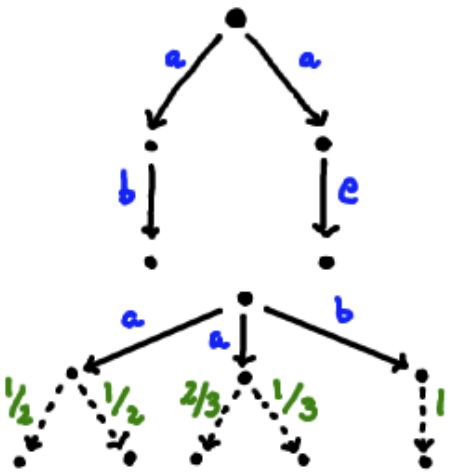
$(S, t : S \to \mathcal{D}_\omega(S) + (A \times S) + 1)$

$(S, t : S \to \mathcal{P}(\mathcal{D}_\omega(\mathcal{P}S)^A))$

What do this things have in common?

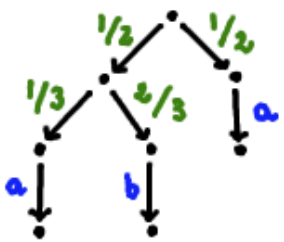


$(S, t : S \to 2 \times S^A)$

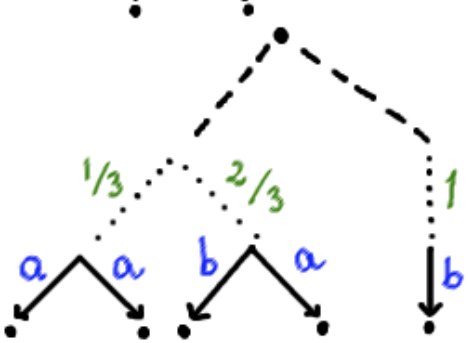


$(S, t : S \to \mathcal{P}S^A)$

$(S, t : S \to \mathcal{P}\mathcal{D}_\omega(S)^A)$



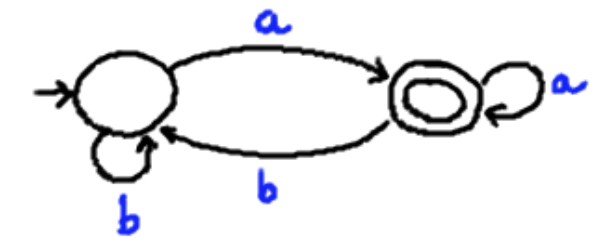
$(S, t : S \to \mathcal{D}_\omega(S) + (A \times S) + 1)$



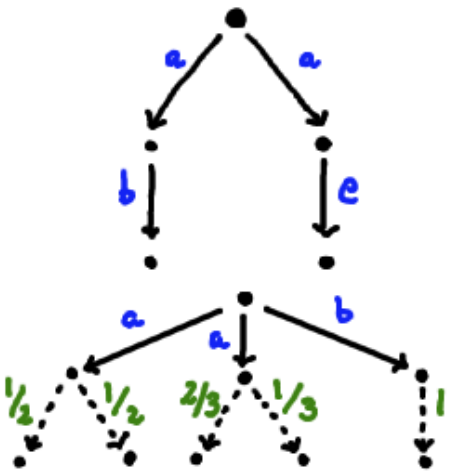
$(S, t : S \to \mathcal{P}(\mathcal{D}_\omega(\mathcal{P}S)^A))$

$(S, t : S \to TS)$

What do this things have in common?

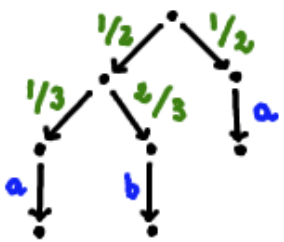


$(S, t : S \to 2 \times S^A)$

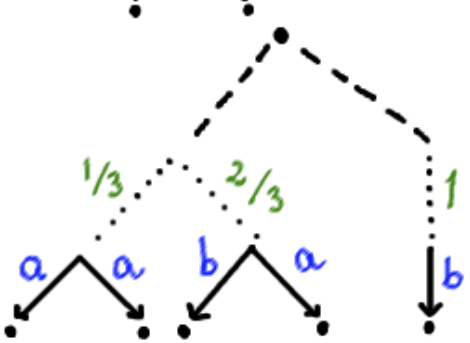


$(S, t : S \to \mathcal{P}S^A)$

$(S, t : S \to \mathcal{P}\mathcal{D}_\omega(S)^A)$



$(S, t : S \to \mathcal{D}_\omega(S) + (A \times S) + 1)$



$(S, t : S \to \mathcal{P}(\mathcal{D}_\omega(\mathcal{P}S)^A))$

$(S, t : S \to TS)$ T-coalgebras

The power of T

$$(S, t : S \to TS)$$

The power of T

$$(S, t : S \to TS)$$

The functor T determines:

1. notion of observational equivalence (coalg. bisimulation)
   E.g. $T = 2 \times (-)^A$: language equivalence

The power of T

$$(S, t : S \to TS)$$

The functor T determines:

1. notion of observational equivalence (coalg. bisimulation)
   E.g. $T = 2 \times (-)^A$: language equivalence
2. behaviour (final coalgebra)
   E.g. $T = 2 \times (-)^A$: languages over A – 2^{A^*}

The power of T

$$(S, t : S \to TS)$$

The functor T determines:

1. notion of observational equivalence (coalg. bisimulation)
   E.g. $T = 2 \times (-)^A$: language equivalence
2. behaviour (final coalgebra)
   E.g. $T = 2 \times (-)^A$: languages over A – 2^{A^*}
3. set of expressions describing finite systems
4. axioms to prove bisimulation equivalence of expressions

1 + 2 are classic coalgebra; 3 + 4 are recent work.

How about algorithms?

- Coalgebra has found its place in the semantic side of the world: operational/denotational semantics, logics, ...
- Are there also opportunities for contributions in algorithms?

How about algorithms?

- Coalgebra has found its place in the semantic side of the world: operational/denotational semantics, logics, ...
- Are there also opportunities for contributions in algorithms?

Brzozowski's algorithm (co)algebraically

Motivation

- duality between reachability and observability (Arbib and Manes 1975): beautiful, not very well-known.
- combined use of algebra and coalgebra.
- our understanding of automata is still very limited;
  cf. recent research: universal automata, àtomata, weighted automata (Sakarovitch, Brzozowski, . . .)

Credits

Bonchi, Bonsangue, Rutten

Credits

Bonchi, Bonsangue, Rutten

It all started with. . .

Prakash Panangaden

Credits

Bonchi, Bonsangue, Rutten

It all started with. . .

Nick Bezhanishvili & Clemens Kupke

Prakash Panangaden

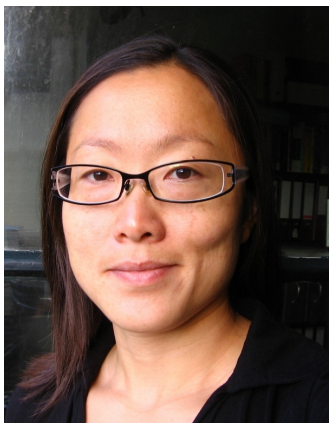
Credits

Bonchi, Bonsangue, Rutten

It all started with. . .

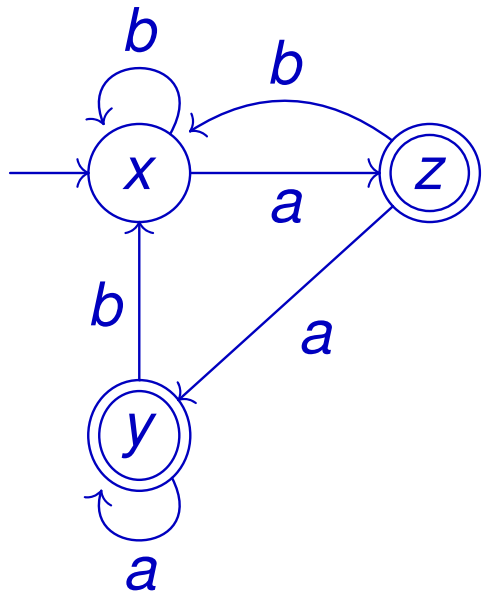
Prakash Panangaden

Nick Bezhanishvili & Clemens Kupke



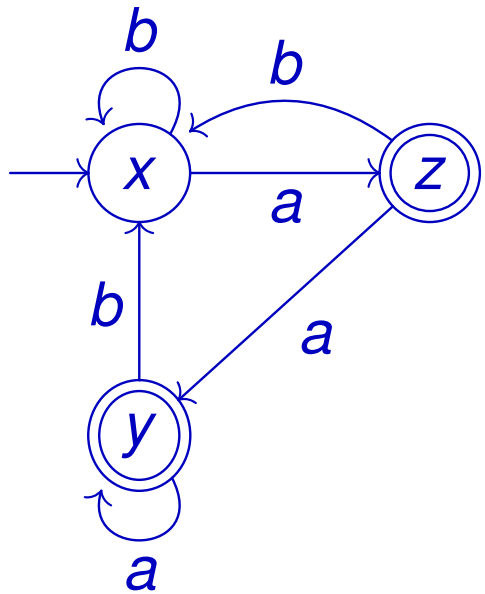
Helle Hansen & Dexter Kozen

Brzozowski algorithm (by example)



- initial state: x
- final states: y and z
- $L(x) = \{a, b\}^* a$

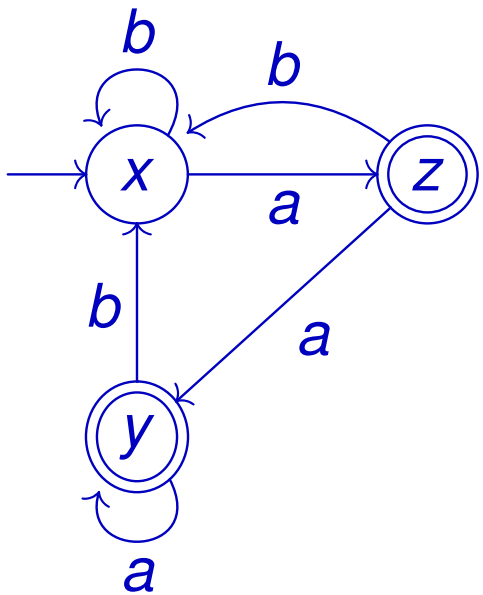
Brzozowski algorithm (by example)



- initial state: x
- final states: y and z
- $L(x) = \{a, b\}^* a$
- X is reachable but not minimal: $L(y) = \varepsilon + \{a, b\}^* a = L(z)$

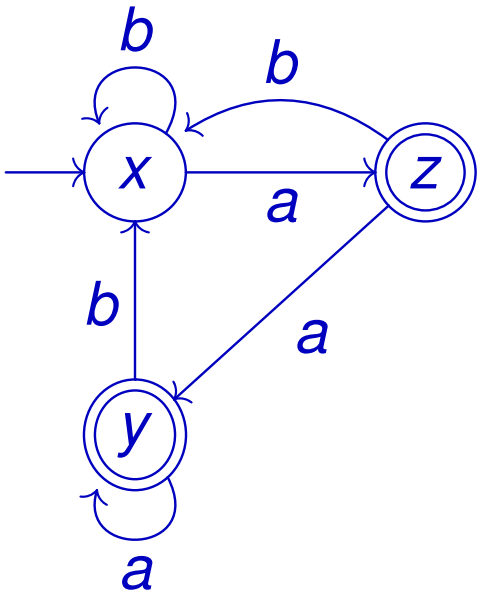
Reversing the automaton: $rev(X)$

$X =$

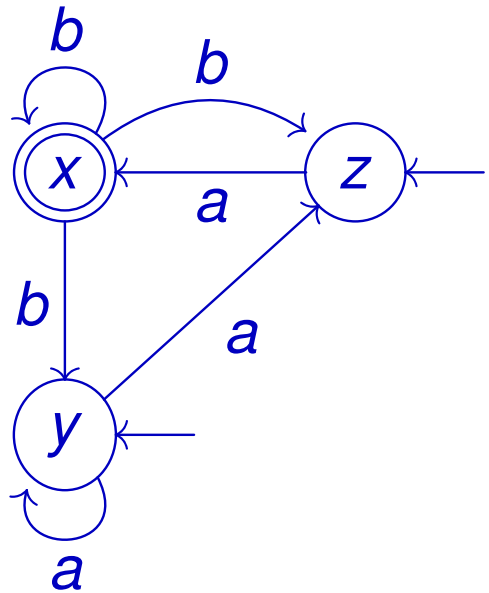


Reversing the automaton: $rev(X)$

$X =$

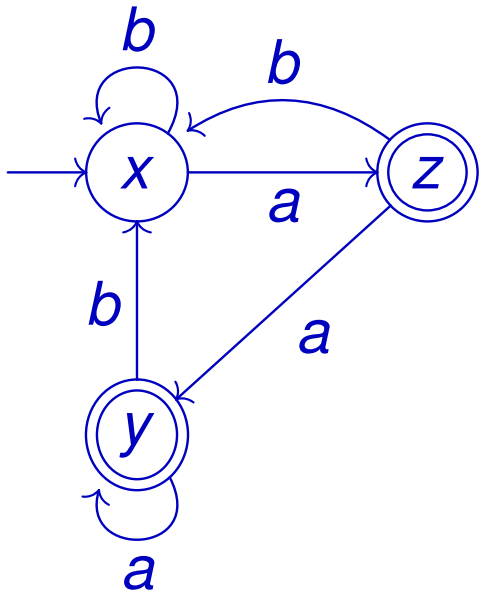


$rev(X) =$

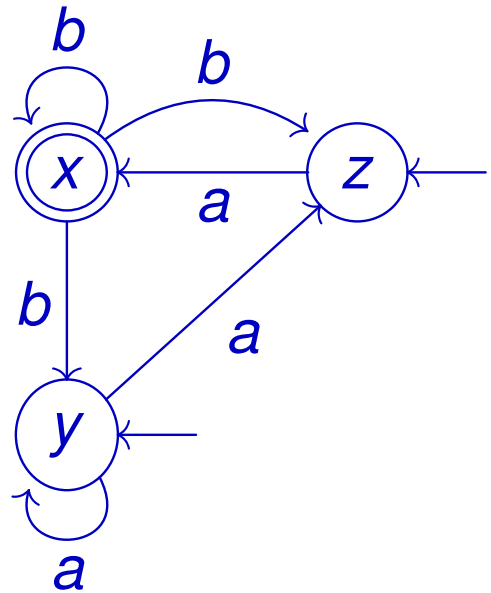


Reversing the automaton: $rev(X)$

$X =$



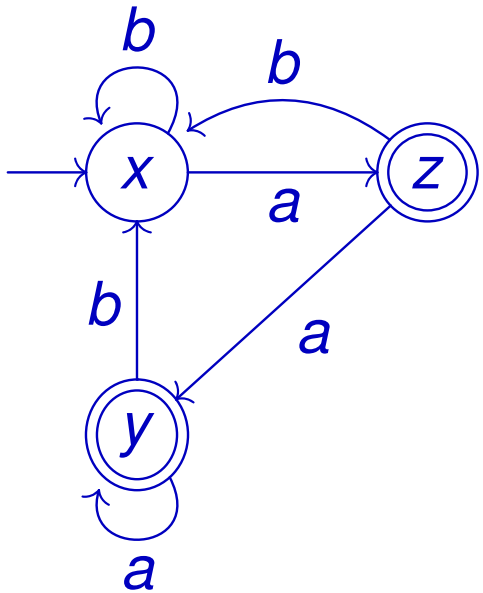
$rev(X) =$



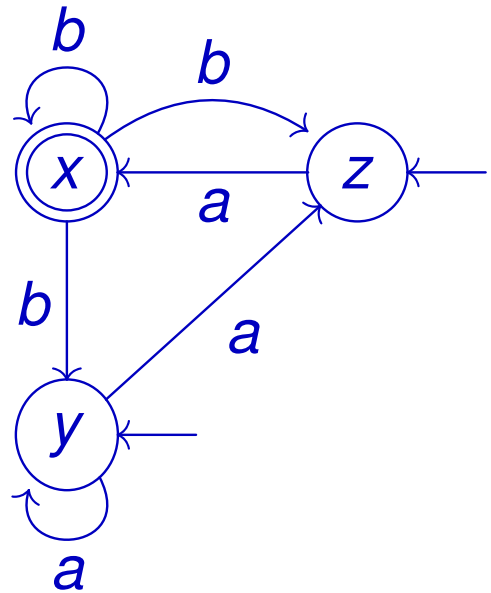
- transitions are reversed
- initial states $\Leftrightarrow$ final states

Reversing the automaton: $rev(X)$

$X =$

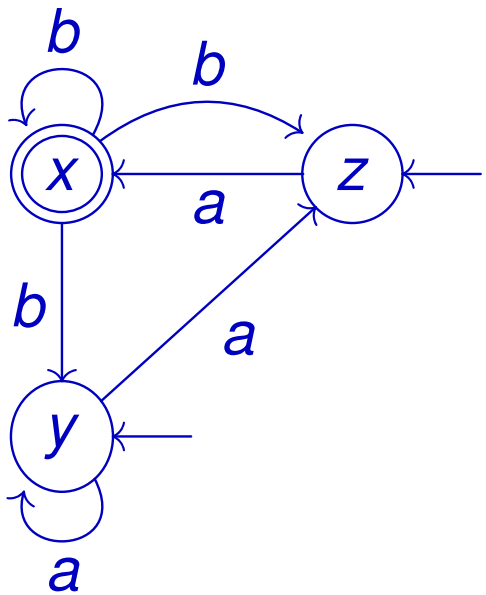


$rev(X) =$

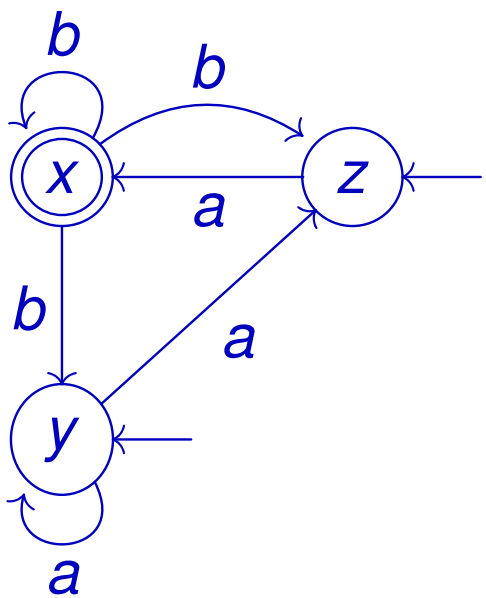


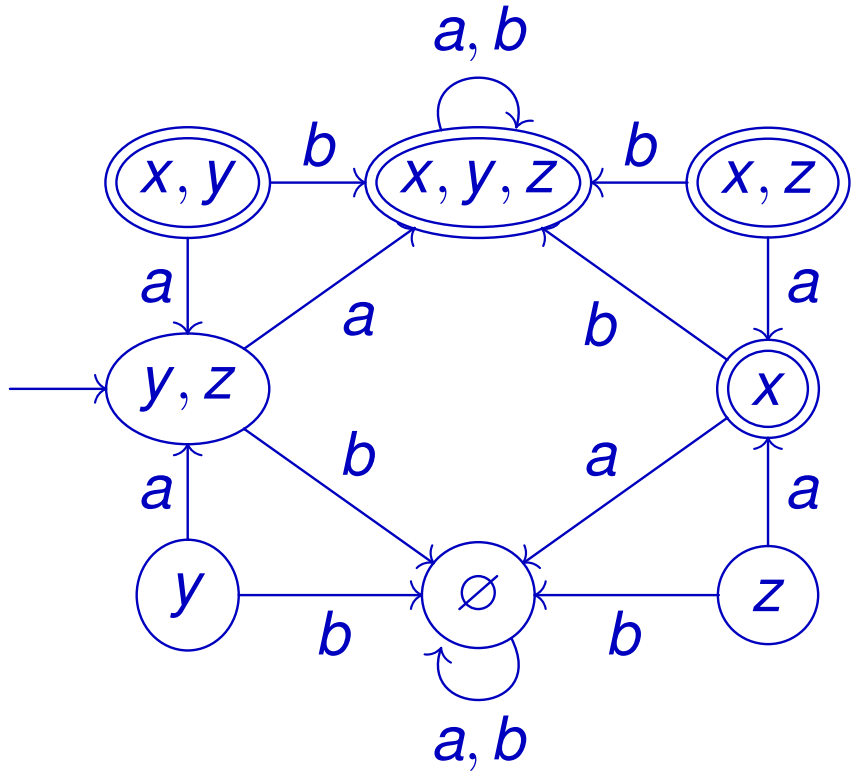
- transitions are reversed
- initial states $\Leftrightarrow$ final states
- $rev(X)$ is non-deterministic

Making it deterministic again: $det(rev(X))$

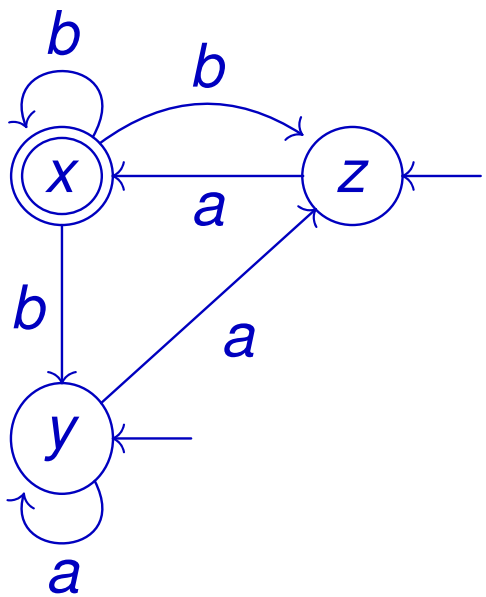


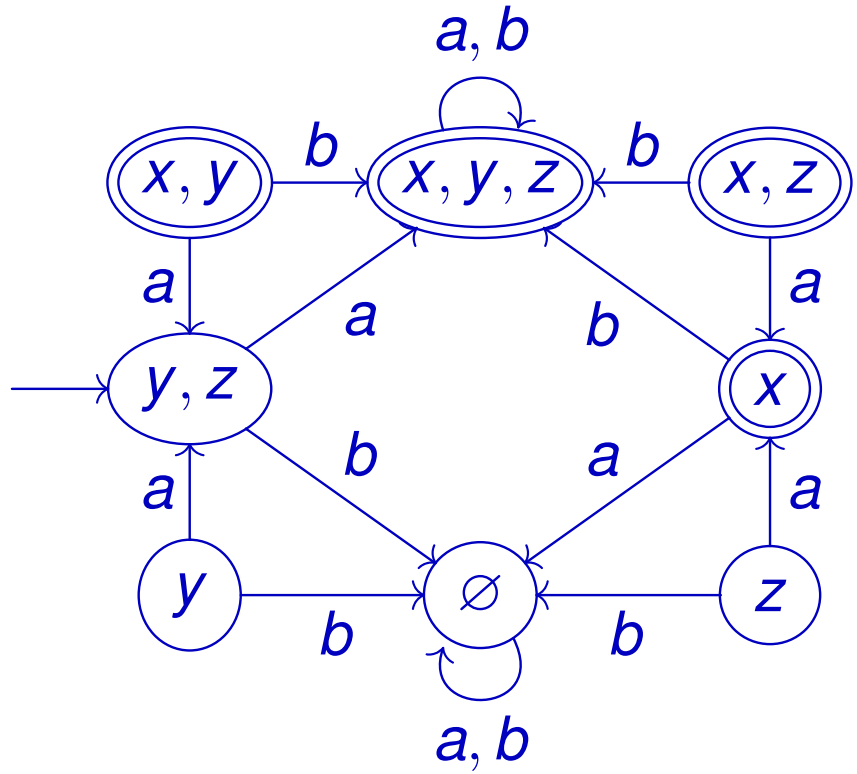
Making it deterministic again: $det(rev(X))$





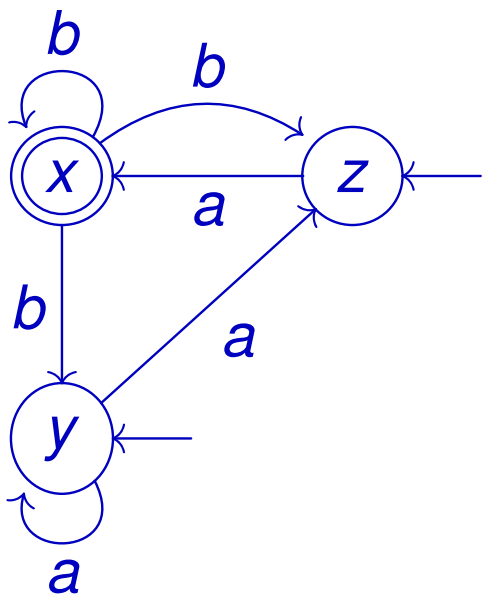
Making it deterministic again: $det(rev(X))$

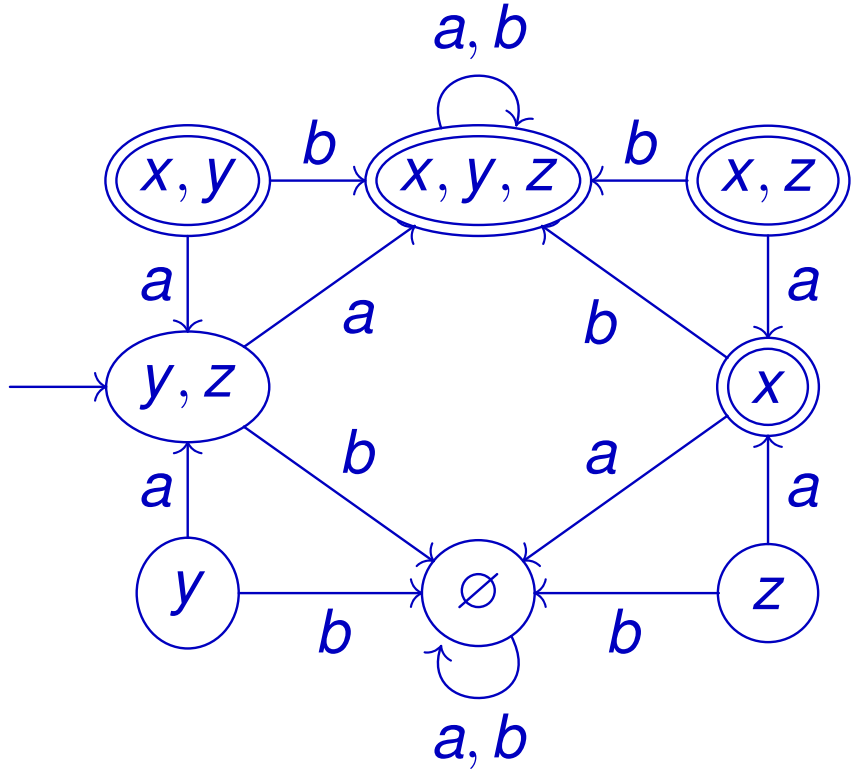




- new state space: $2^X = \{V \mid V \subseteq \{x, y, z\}\}$

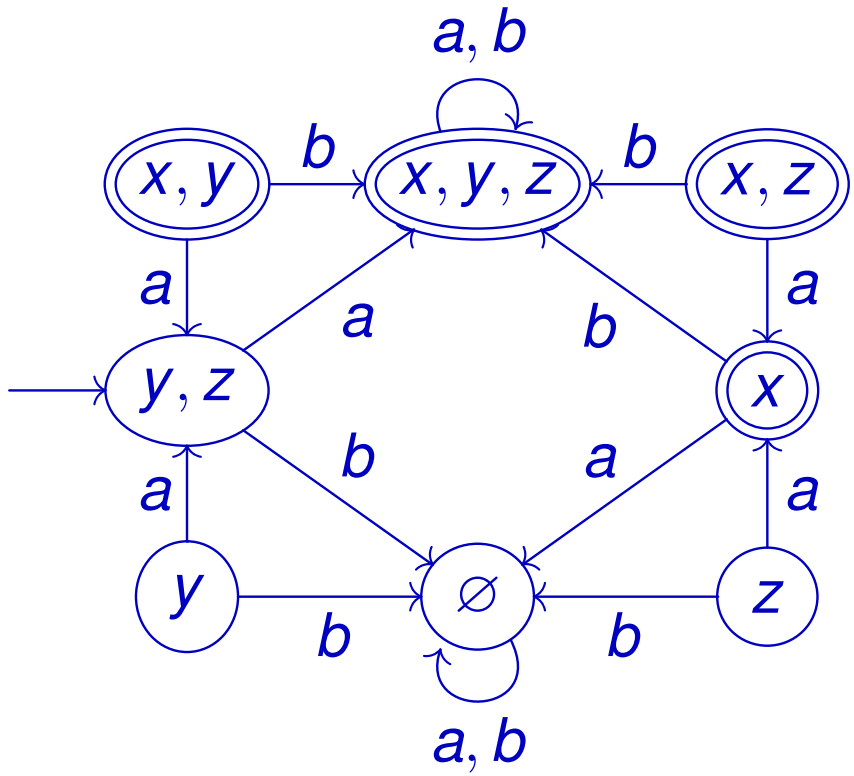
Making it deterministic again: $det(rev(X))$



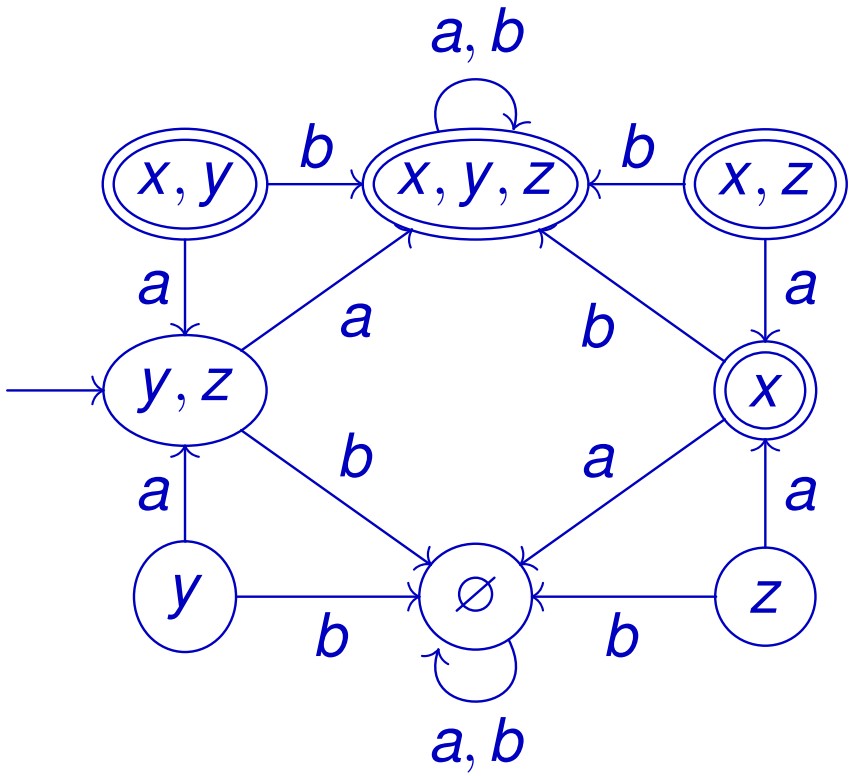


- new state space: $2^X = \{V \mid V \subseteq \{x, y, z\}\}$
- initial state: $\{y, z\}$ final states: all V with $x \in V$
- $V \xrightarrow{a} W$ $W = \{w \mid v \xrightarrow{a} w, v \in V\}$

The automaton $det(rev(X))$ . . .



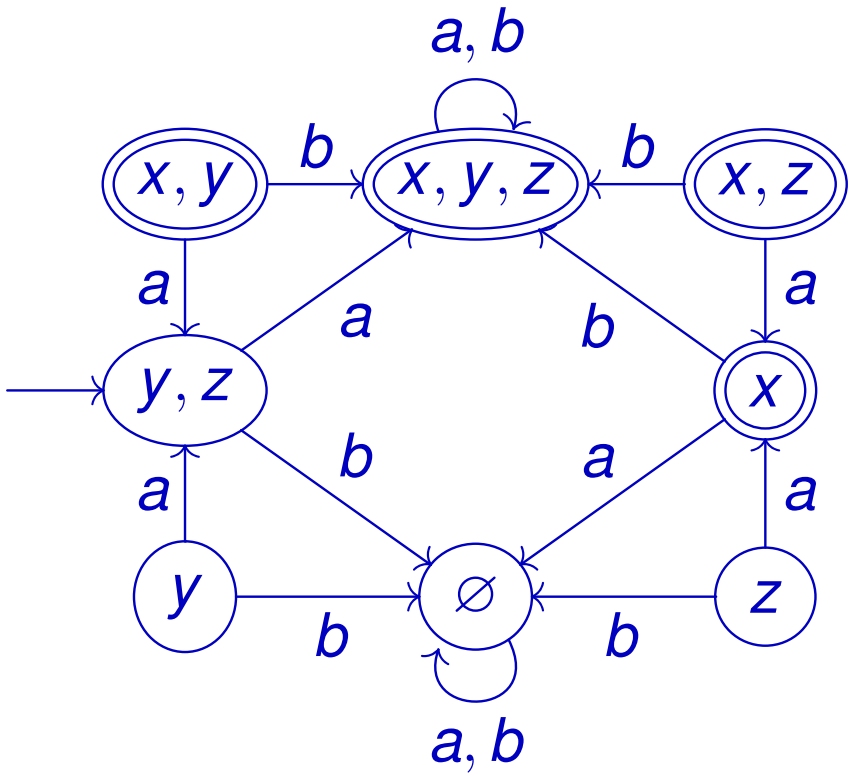
The automaton $det(rev(X))$ . . .



- . . . accepts the reverse of the language accepted by X:

$$L(\, det(rev(X))\,) \;=\; a\{a,b\}^* \;=\; reverse(\, L(X)\,)$$

The automaton $det(rev(X))$ . . .



- . . . accepts the reverse of the language accepted by X:

$$L(\, det(rev(X))\,) \;=\; a\{a,b\}^* \;=\; reverse(\, L(X)\,)$$

- . . . and is observable!

Today's Theorem

If: a deterministic automaton X is reachable and accepts $L(X)$

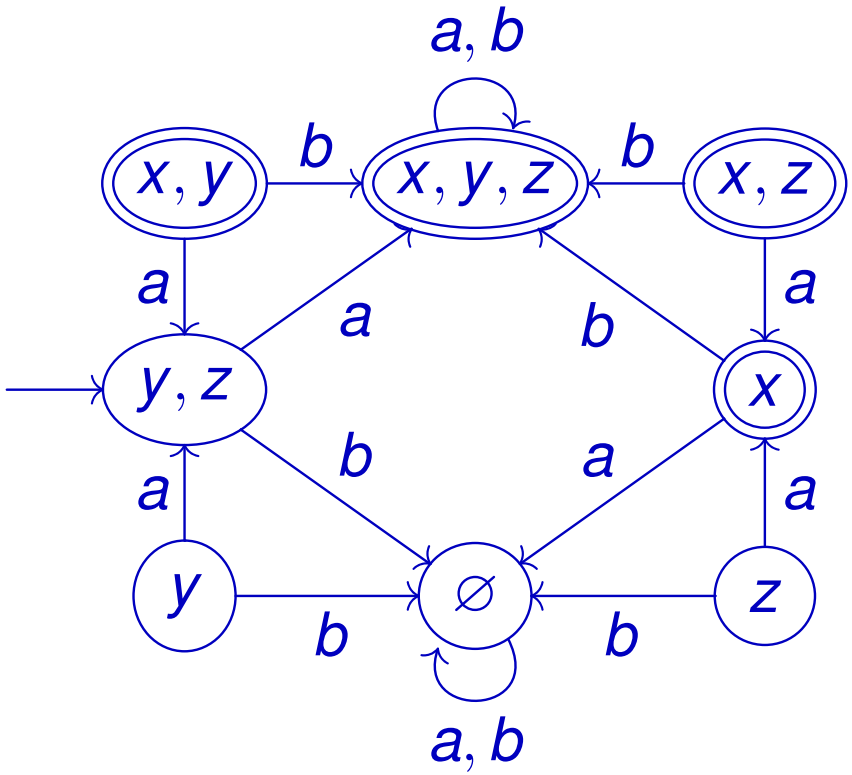
Today's Theorem

If: a deterministic automaton X is reachable and accepts $L(X)$

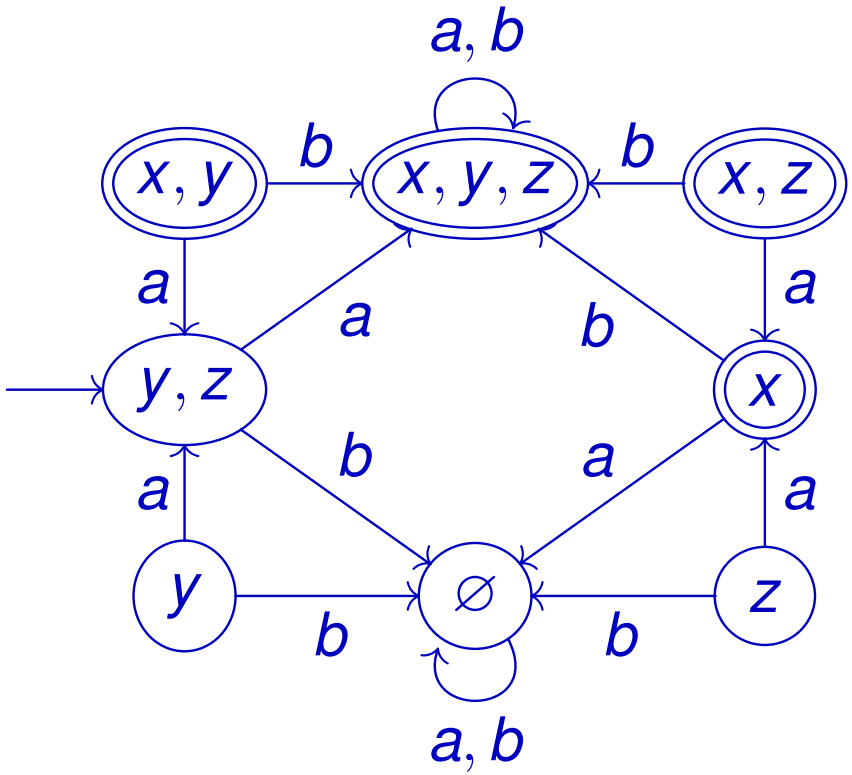
then: $det(rev(X))$ is minimal and

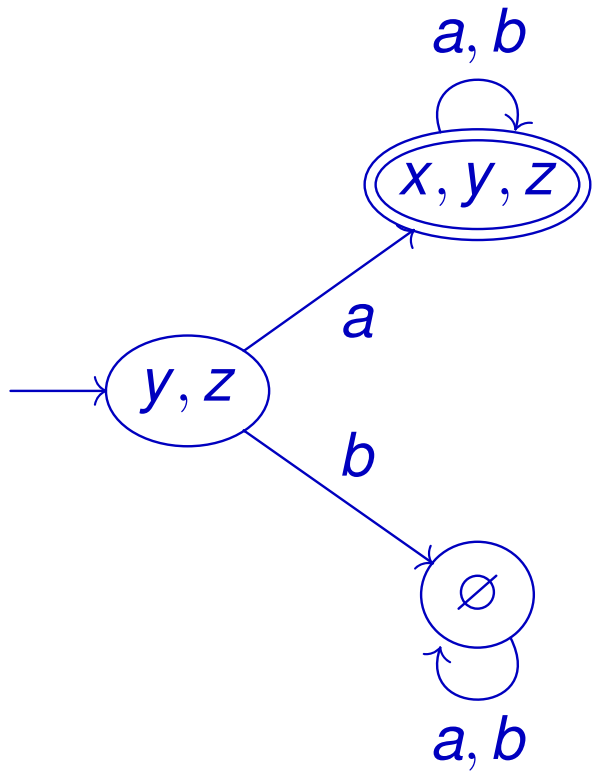
$$L(\, det(rev(X))\,) \;=\; reverse(\, L(X)\,)$$

Taking the reachable part of $det(rev(X))$



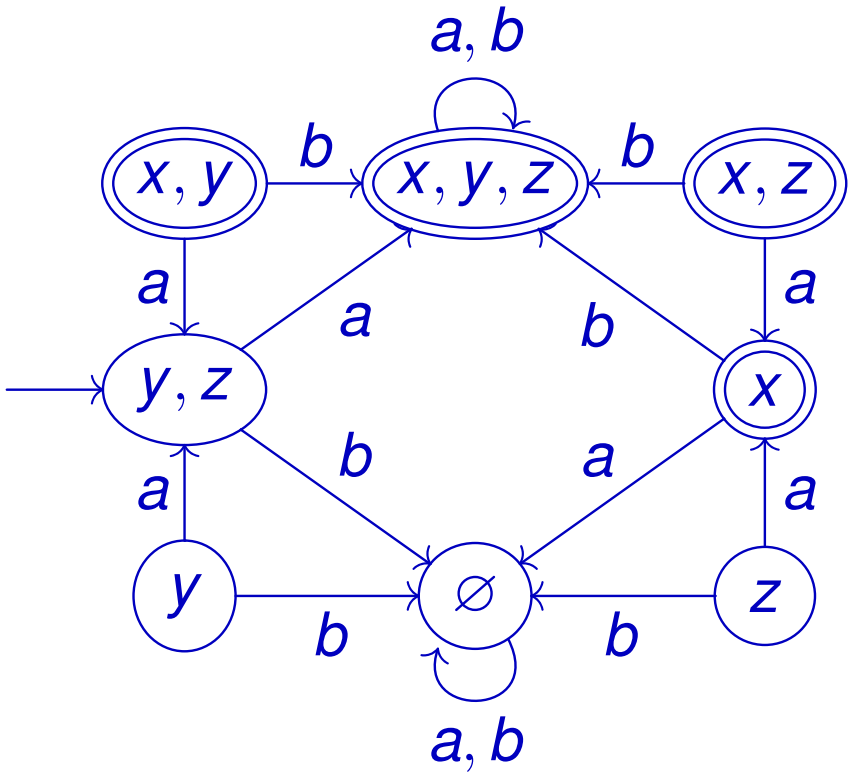
Taking the reachable part of $det(rev(X))$

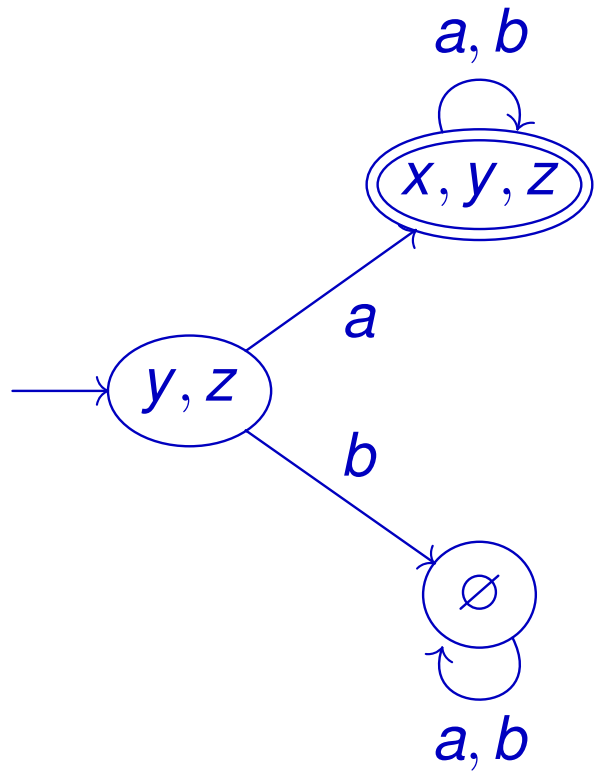




- $reach(det(rev(X)))$

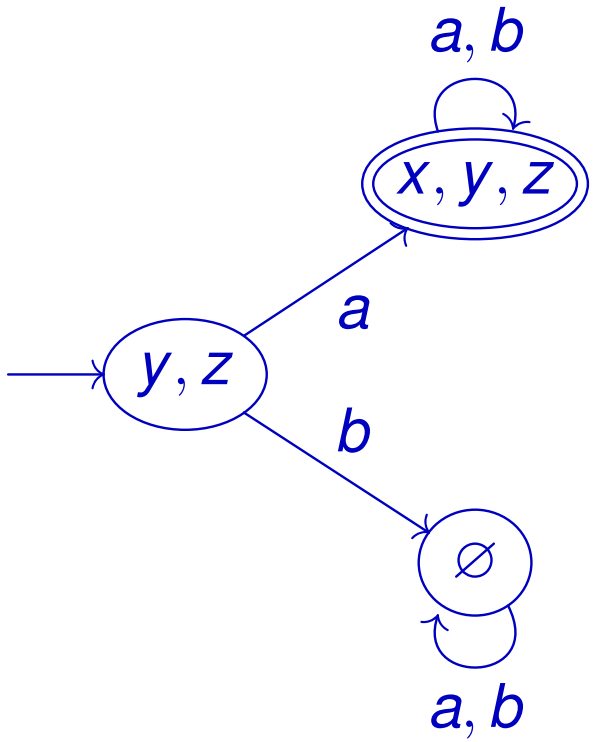
Taking the reachable part of $det(rev(X))$



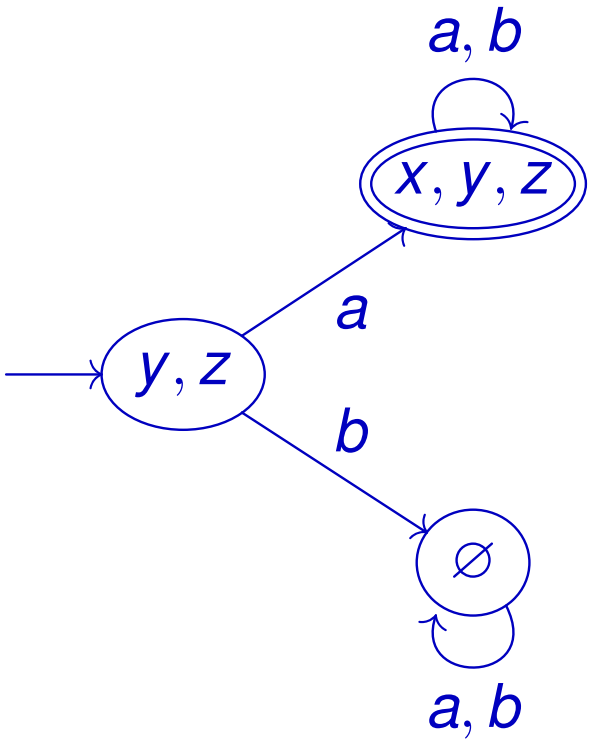


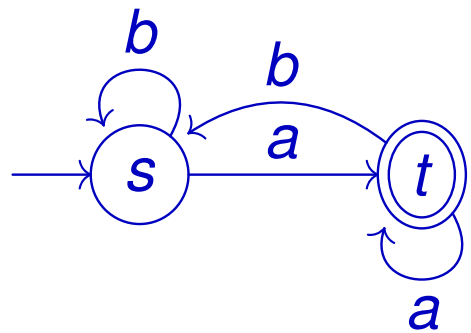
- $reach(det(rev(X)))$ is reachable (by construction)

Repeating everything, now for $reach(det(rev(X)))$

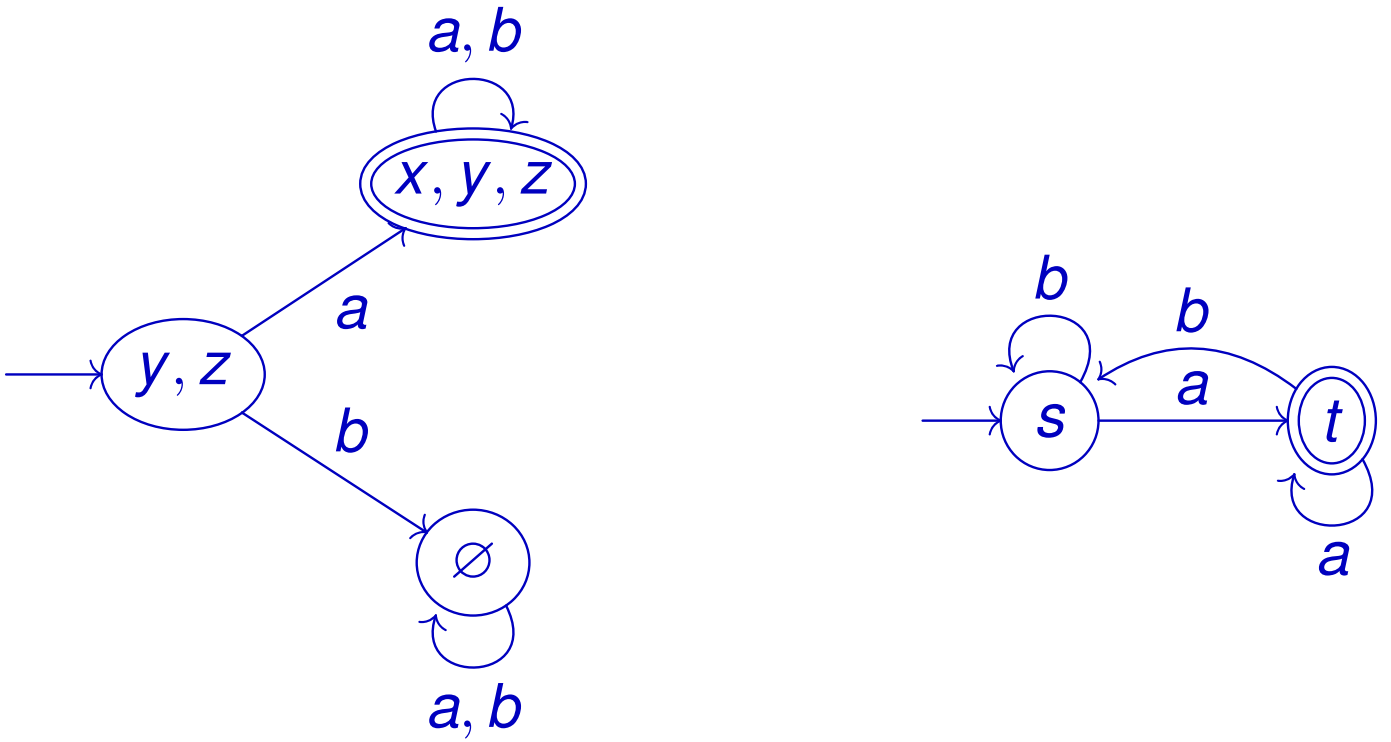


Repeating everything, now for $reach(det(rev(X)))$



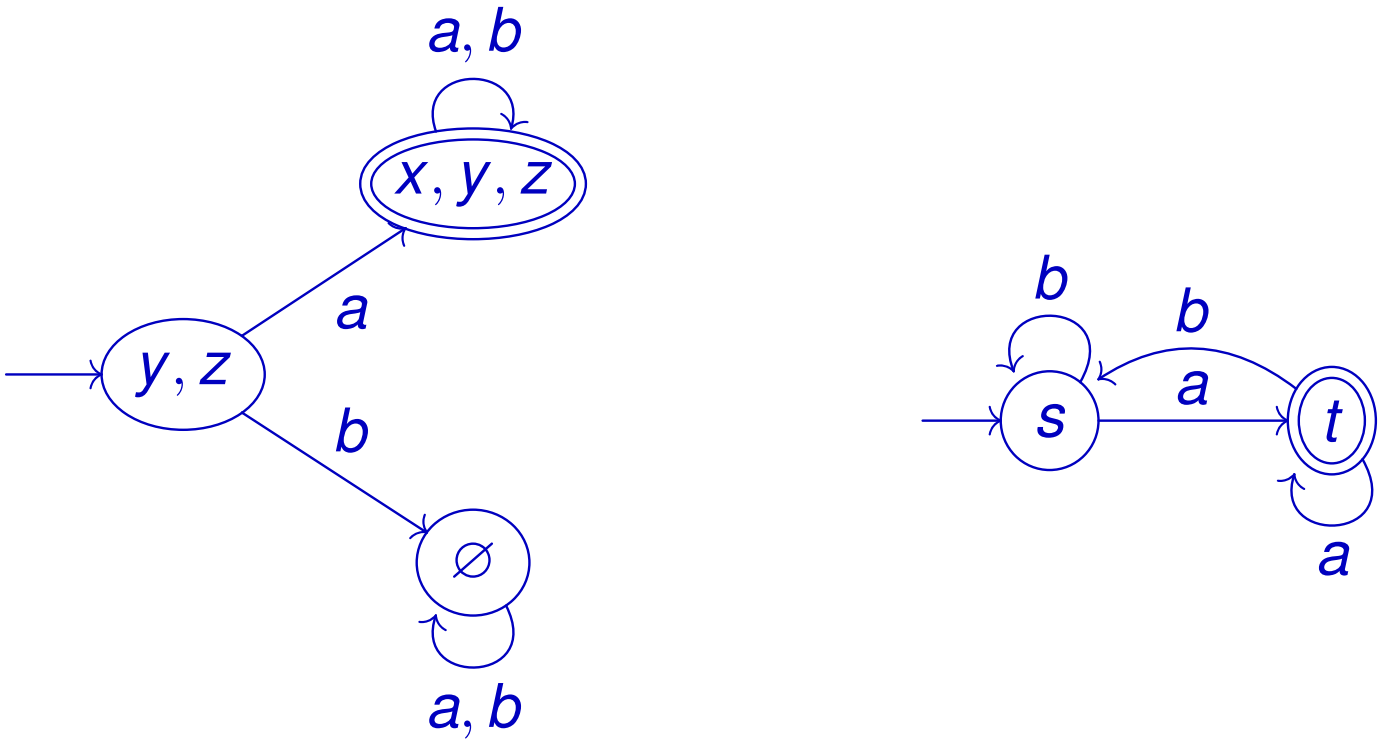


Repeating everything, now for $reach(det(rev(X)))$



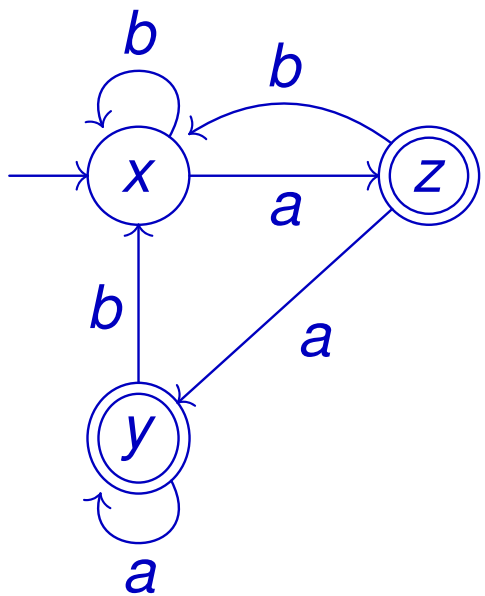
- . . . gives us $reach(det(rev(reach(det(rev(X))))))$

Repeating everything, now for $reach(det(rev(X)))$

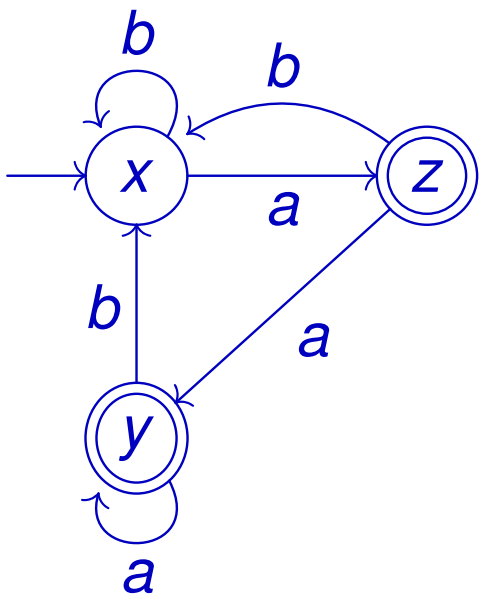


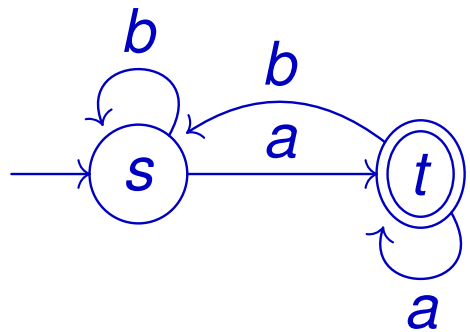
- . . . gives us $reach(det(rev(reach(det(rev(X))))))$
- which is (reachable and) minimal and accepts $\{a, b\}^* a$.

All in all: Brzozowski's algorithm

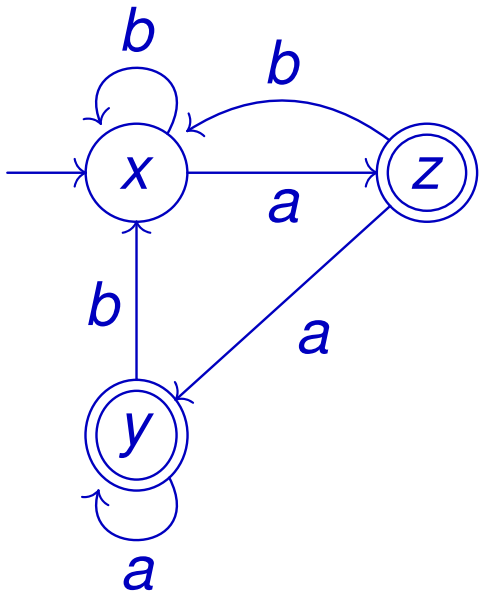


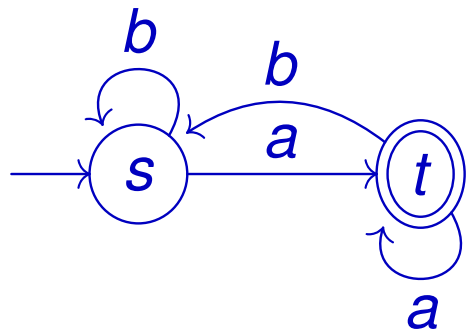
All in all: Brzozowski's algorithm





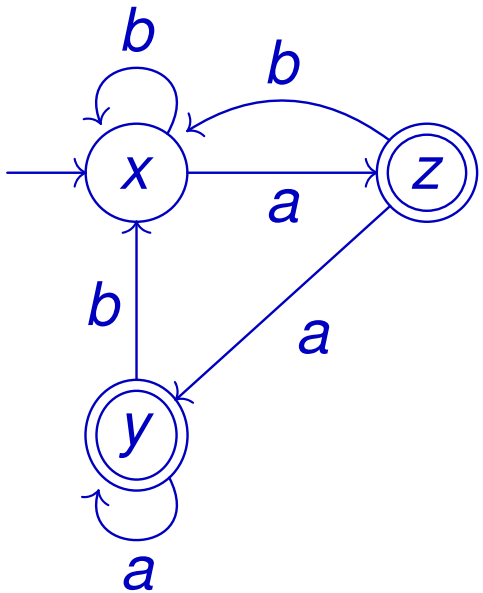
All in all: Brzozowski's algorithm

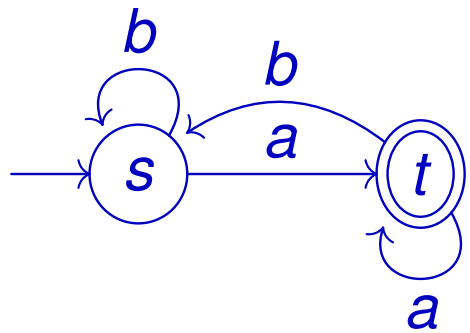




- X is reachable and accepts $\{a, b\}^* a$

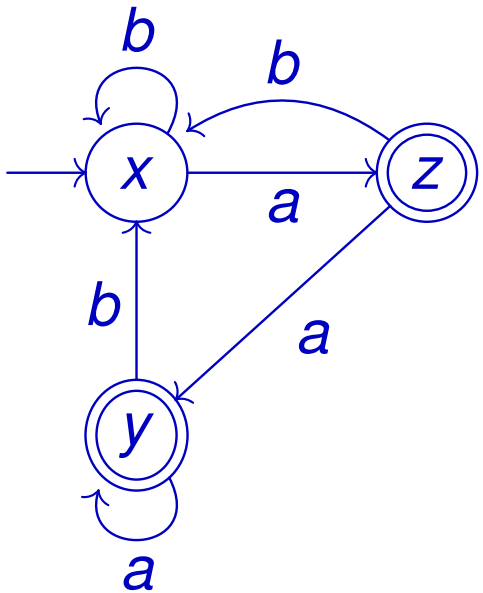
All in all: Brzozowski's algorithm

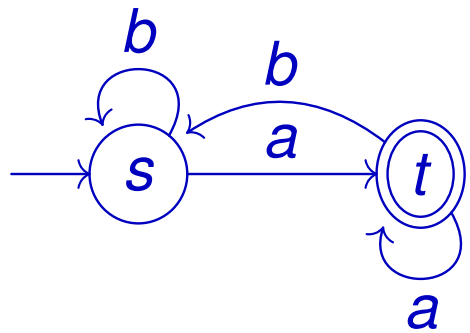




- X is reachable and accepts $\{a, b\}^* a$
- $reach(det(rev(reach(det(rev(X))))))$ also accepts $\{a, b\}^* a$

All in all: Brzozowski's algorithm





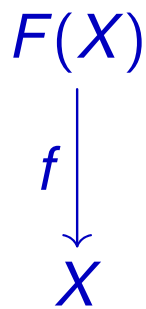
- X is reachable and accepts $\{a, b\}^* a$
- $reach(det(rev(reach(det(rev(X))))))$ also accepts $\{a, b\}^* a$
- . . . and is minimal!!

Goal of the day

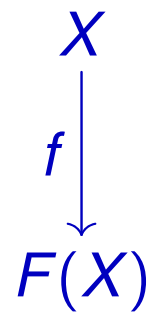
- Correctness of Brzozowski's algorithm (co)algebraically
- Generalizations to other types of automata

(Co)algebra

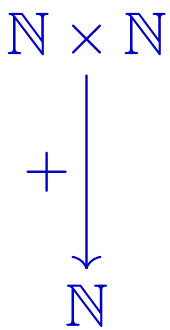
algebras:



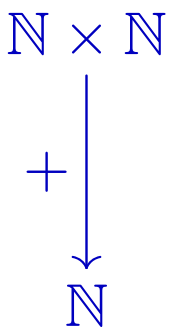
coalgebras:

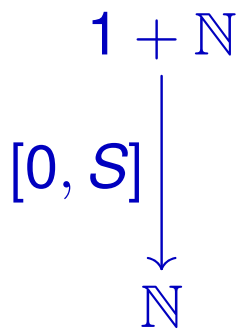


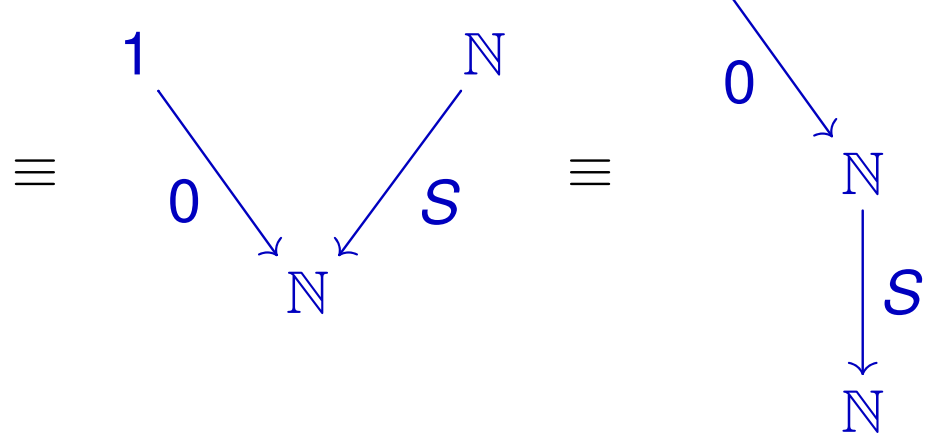
Examples of algebras



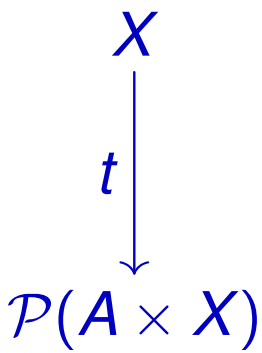
Examples of algebras





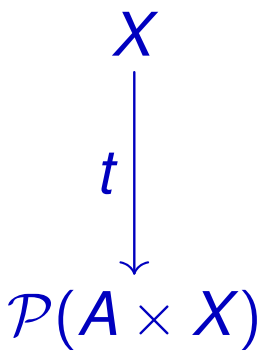


Examples of coalgebras



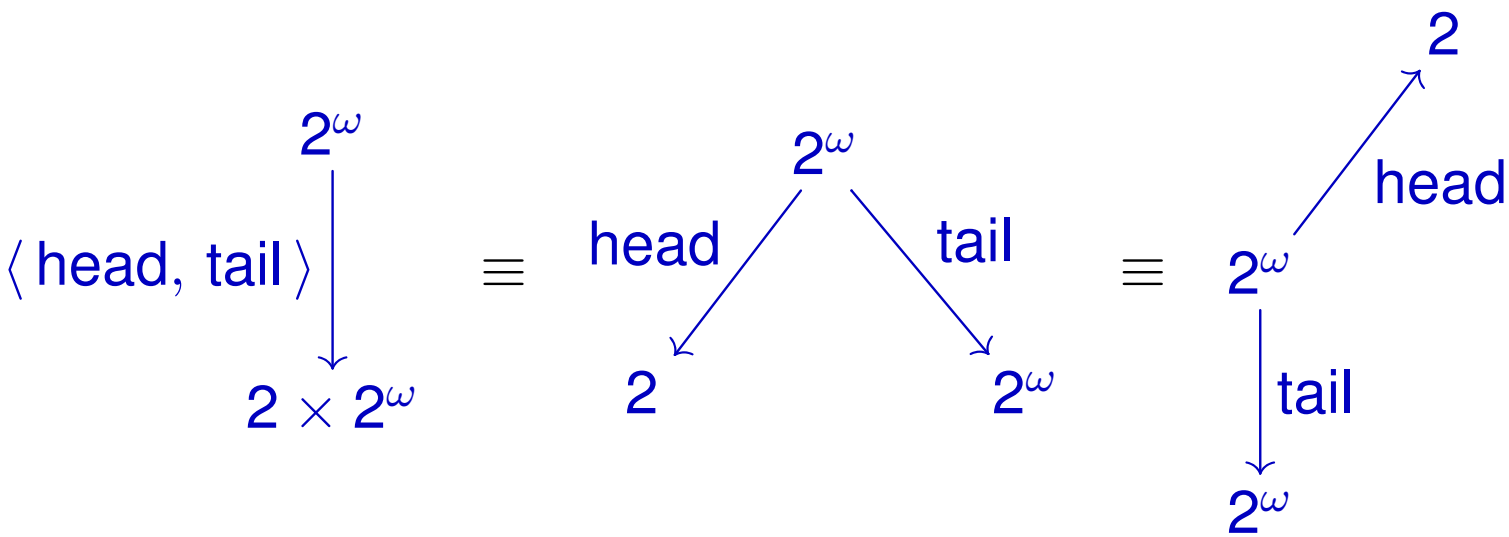
$$x \xrightarrow{a} y \quad \leftrightarrow \quad \langle a, y \rangle \in t(x)$$

Examples of coalgebras



$$x \xrightarrow{a} y \quad \leftrightarrow \quad \langle a, y \rangle \in t(x)$$

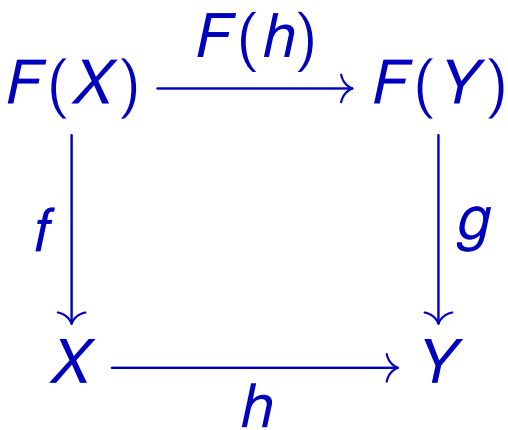
Examples of coalgebras



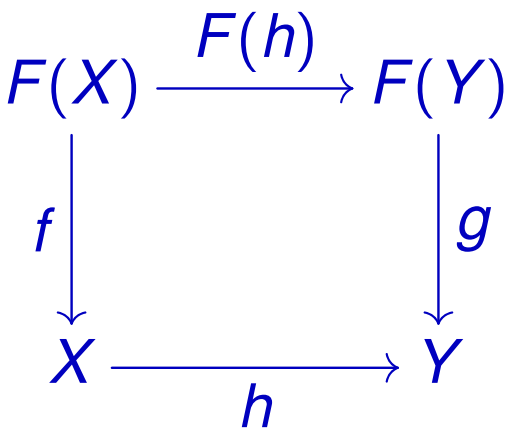
$$\text{head}((b_0, b_1, b_2, \ldots)) = b_0$$

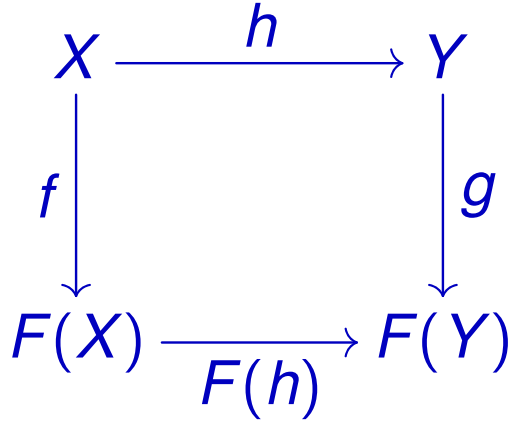
$$\text{tail}((b_0, b_1, b_2, \ldots)) = (b_1, b_2, b_3 \ldots)$$

Homomorphisms

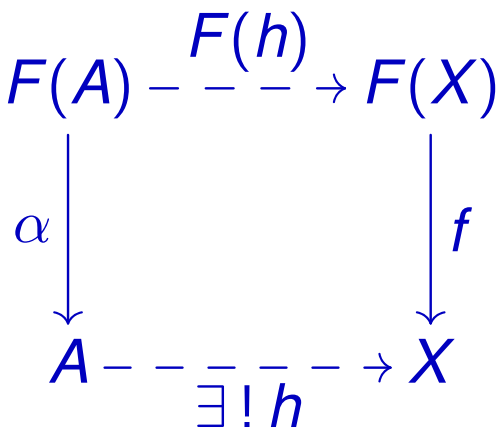


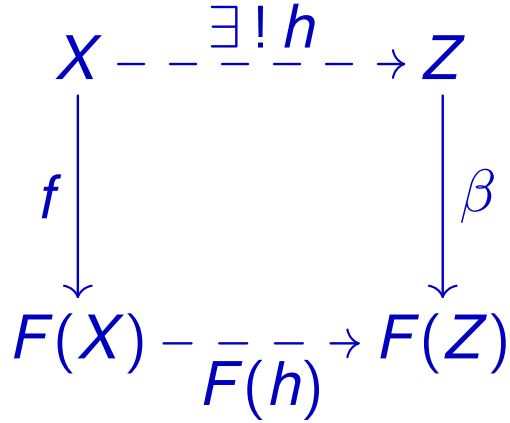
Homomorphisms



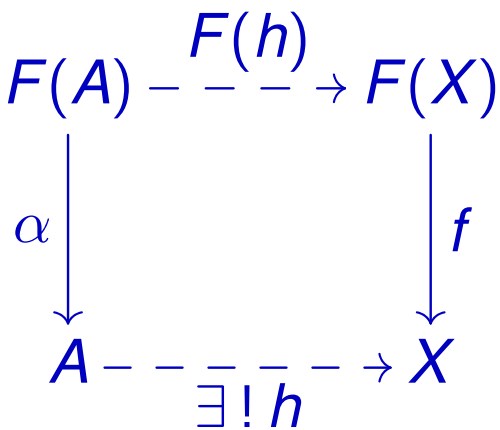


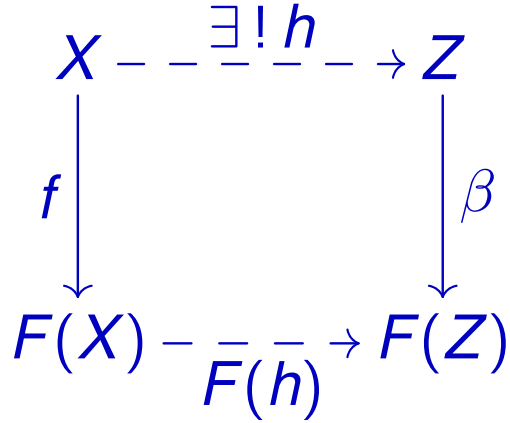
Initiality, finality





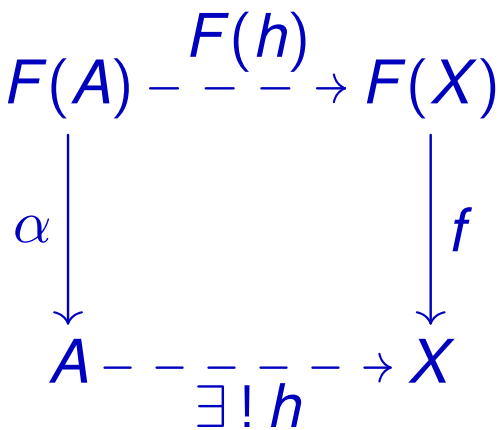
Initiality, finality

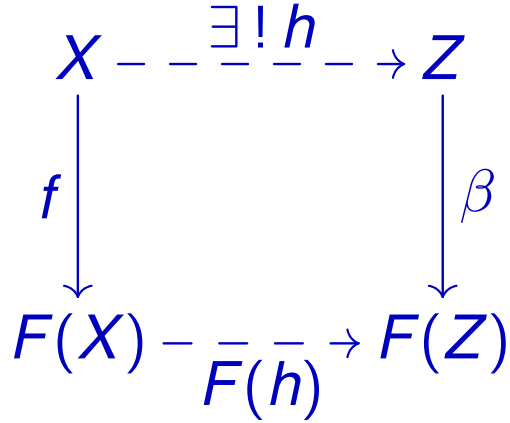




- initial algebras $\leftrightarrow$ induction

Initiality, finality





- initial algebras $\leftrightarrow$ induction
- final coalgebras $\leftrightarrow$ coinduction

Automata, (co)algebraically

- Automata are complicated structures:

  part of them is algebra - part of them is coalgebra

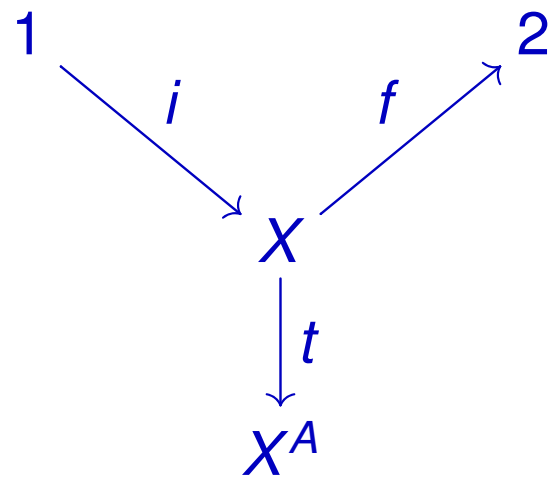
Automata, (co)algebraically

- Automata are complicated structures:

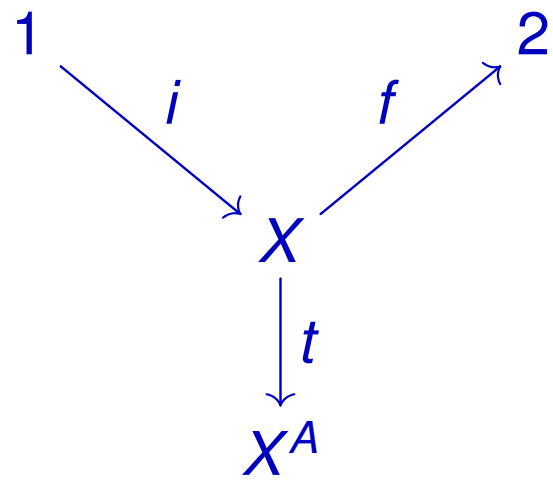
  part of them is algebra - part of them is coalgebra

- (. . . in two different ways . . .)

A deterministic automaton

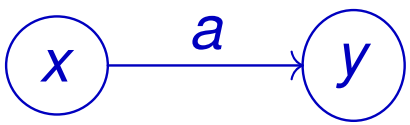


A deterministic automaton



where

$1 = \{0\}$ $\quad 2 = \{0, 1\}$ $\quad X^A = \{\, g \mid g : A \to X \,\}$



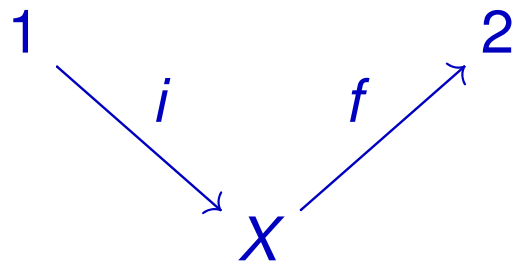
$\leftrightarrow \quad t(x)(a) = y$

$i(0) \in X$ is the initial state

is final (or accepting) $\leftrightarrow \quad f(x) = 1$

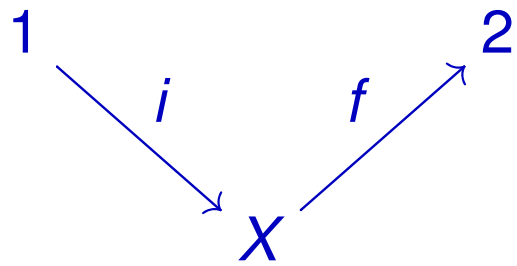
Automata: algebra or coalgebra?

- initial state: algebraic – final states: coalgebraic



Automata: algebra or coalgebra?

- initial state: algebraic – final states: coalgebraic



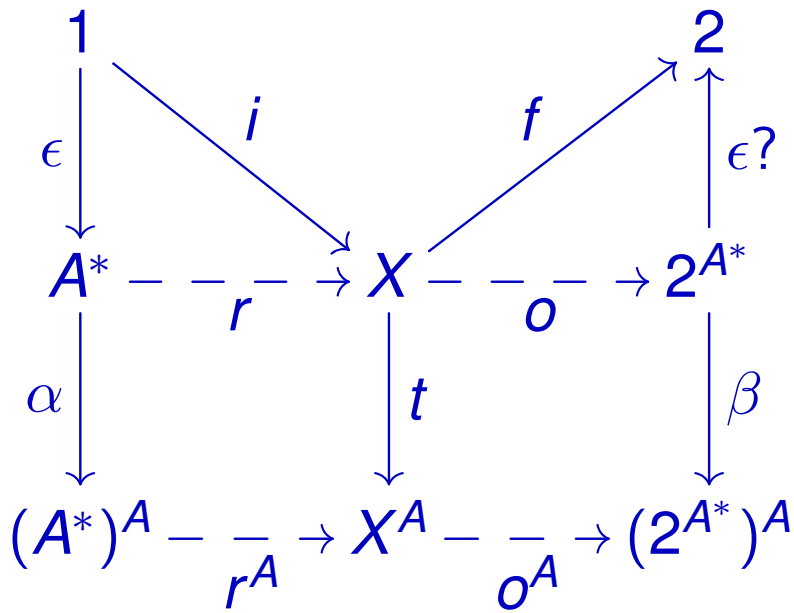
- transition function: both algebraic and coalgebraic

$$\frac{X \xrightarrow{t} X^A}{}$$

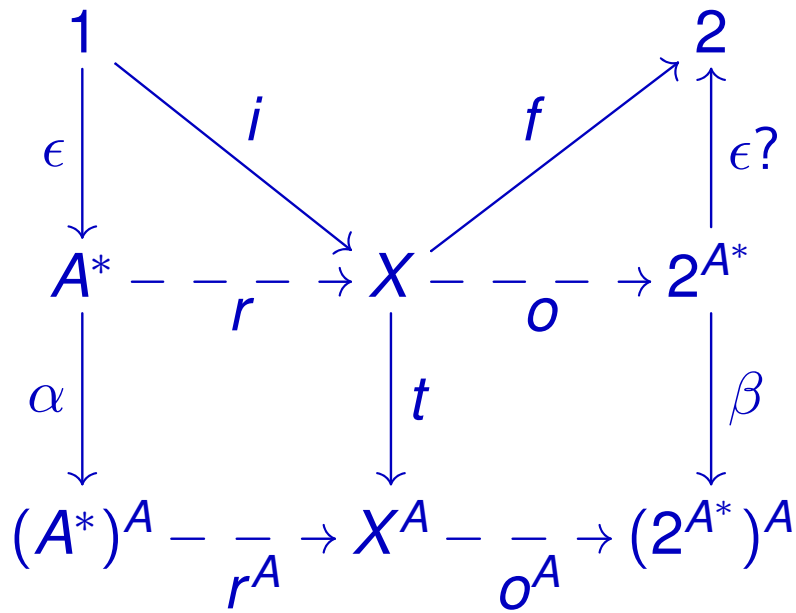
$$\frac{X \longrightarrow (A \longrightarrow X)}{}$$

$$X \times A \xrightarrow{t} X$$

Automata: algebra and coalgebra!

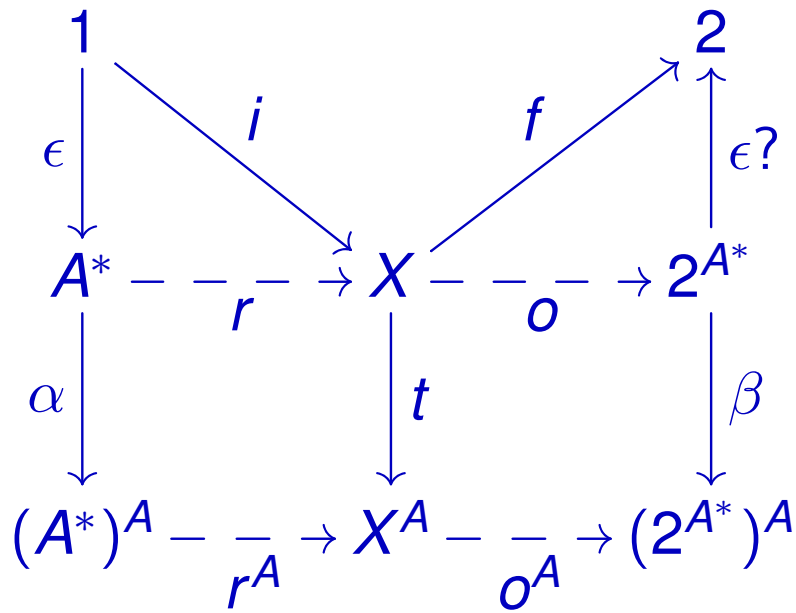


Automata: algebra and coalgebra!



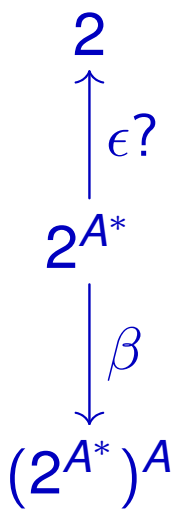
To take home: this picture!! . . .

Automata: algebra and coalgebra!



To take home: this picture!! . . . which we'll explain next . . .

The “automaton” of languages

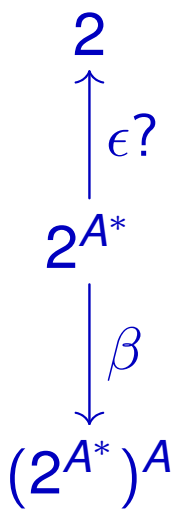


$$\epsilon?(L) = 1 \leftrightarrow \epsilon \in L$$

$$2^{A^*} = \{g \mid g : A^* \to 2\} \cong \{L \mid L \subseteq A^*\}$$

$$\beta(L)(a) = L_a = \{w \in A^* \mid a \cdot w \in L\}$$

The “automaton” of languages



$$\epsilon?(L) = 1 \leftrightarrow \epsilon \in L$$

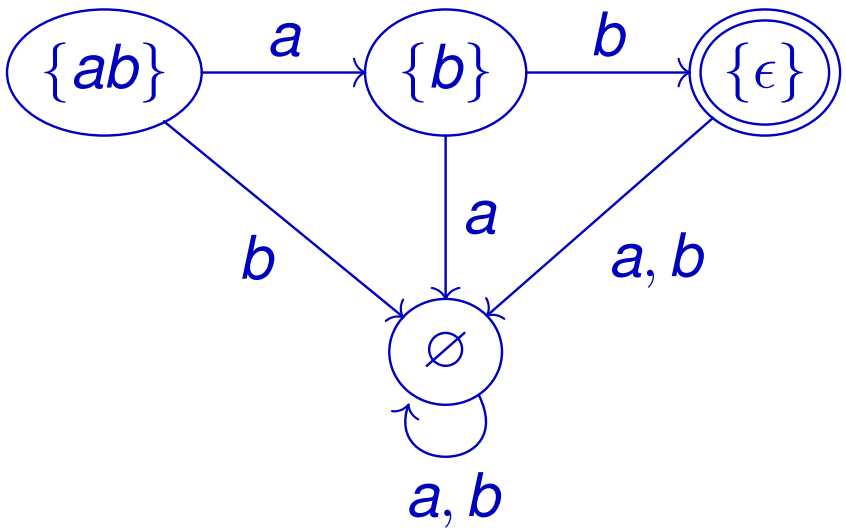
$$2^{A^*} = \{g \mid g : A^* \to 2\} \cong \{L \mid L \subseteq A^*\}$$

$$\beta(L)(a) = L_a = \{w \in A^* \mid a \cdot w \in L\}$$

- We say “automaton”: it does not have an initial state.

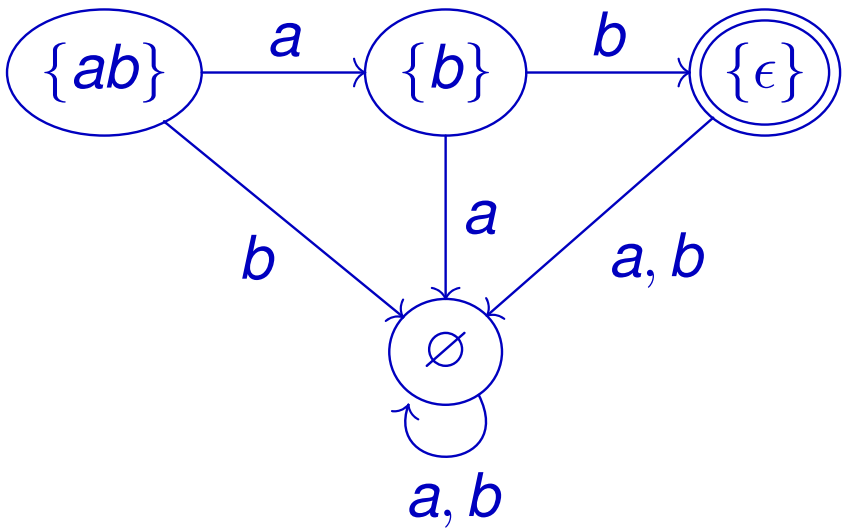
The automaton of languages

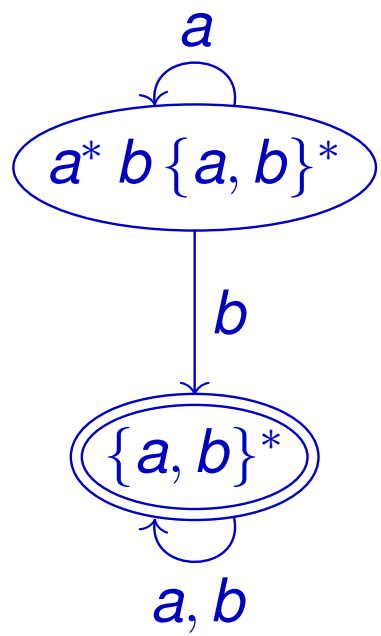
- transitions: $L \xrightarrow{a} L_a$ where $L_a = \{w \in A^* \mid a \cdot w \in L\}$
- for instance:



The automaton of languages

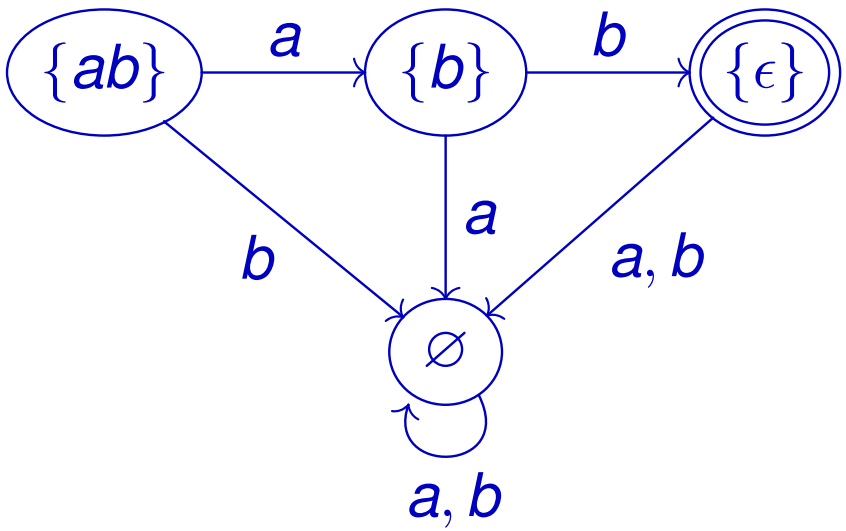
- transitions: $L \xrightarrow{a} L_a$ where $L_a = \{w \in A^* \mid a \cdot w \in L\}$
- for instance:

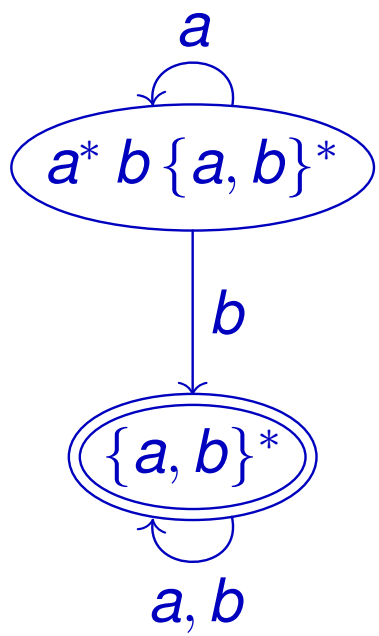




The automaton of languages

- transitions: $L \xrightarrow{a} L_a$ where $L_a = \{w \in A^* \mid a \cdot w \in L\}$
- for instance:

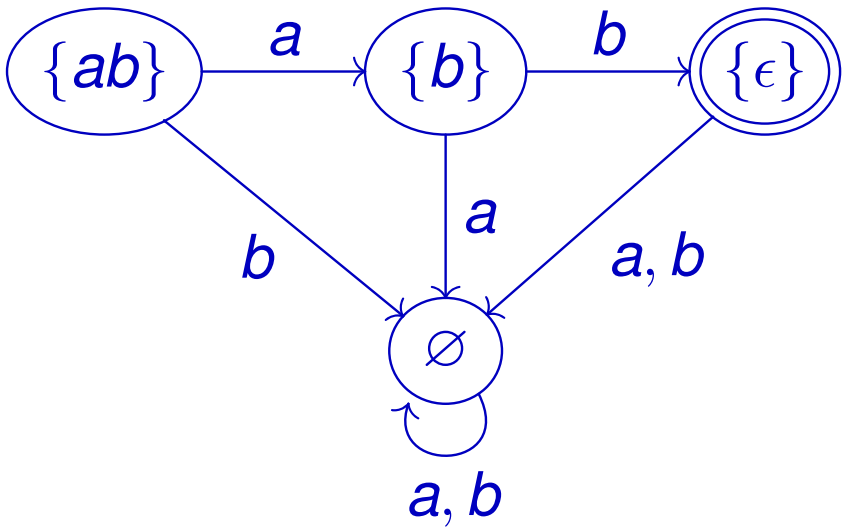


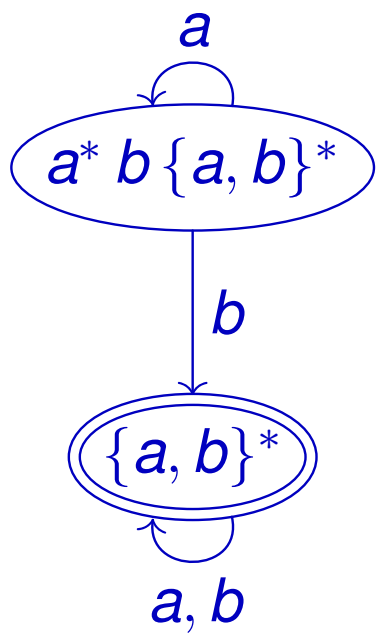


- note: every state L accepts . . .

The automaton of languages

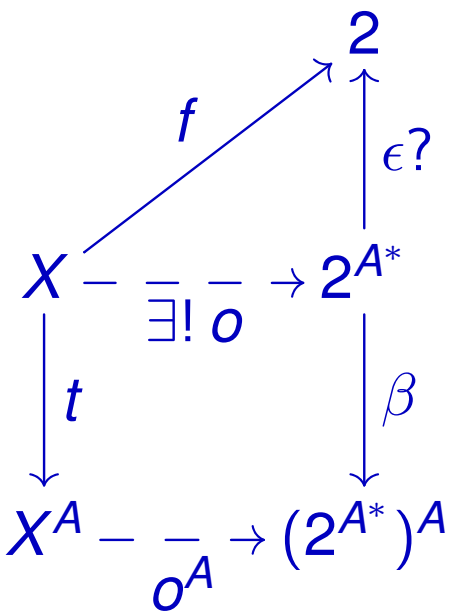
- transitions: $L \xrightarrow{a} L_a$ where $L_a = \{w \in A^* \mid a \cdot w \in L\}$
- for instance:





- note: every state L accepts . . . . . the language L !!

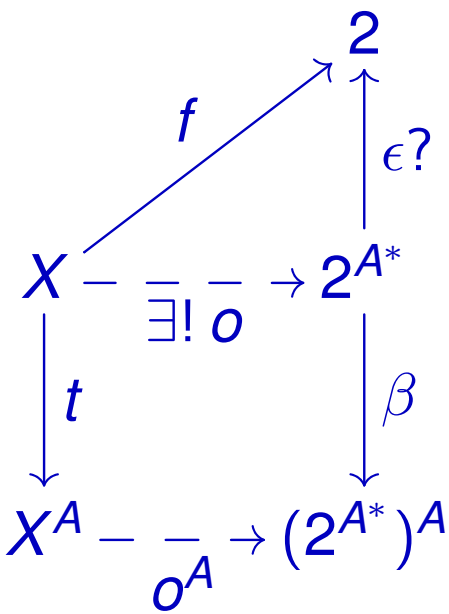
The automaton of languages is . . . final



$$o(x) = \{w \in A^* \mid f(x_w) = 1\}$$

$$= \text{the language accepted by } x$$

The automaton of languages is . . . final



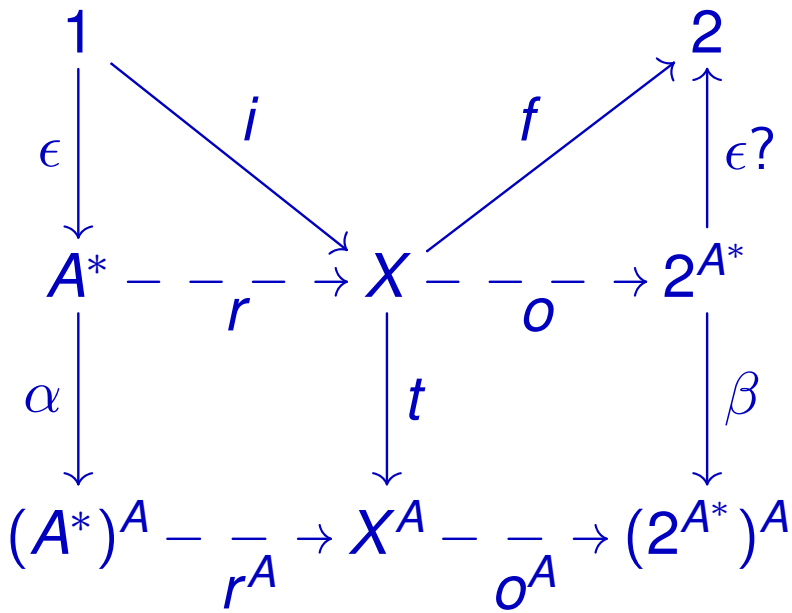
$$o(x) = \{w \in A^* \mid f(x_w) = 1\}$$

$$= \text{the language accepted by } x$$

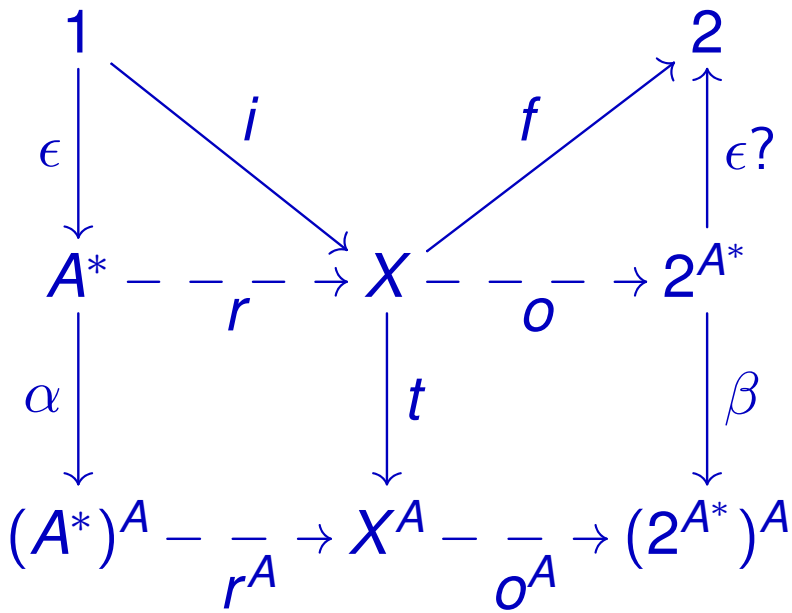
where: x_w is the state reached after inputting the word w,

and: $o^A(g) = o \circ g$, all $g \in X^A$.

Back to today's picture

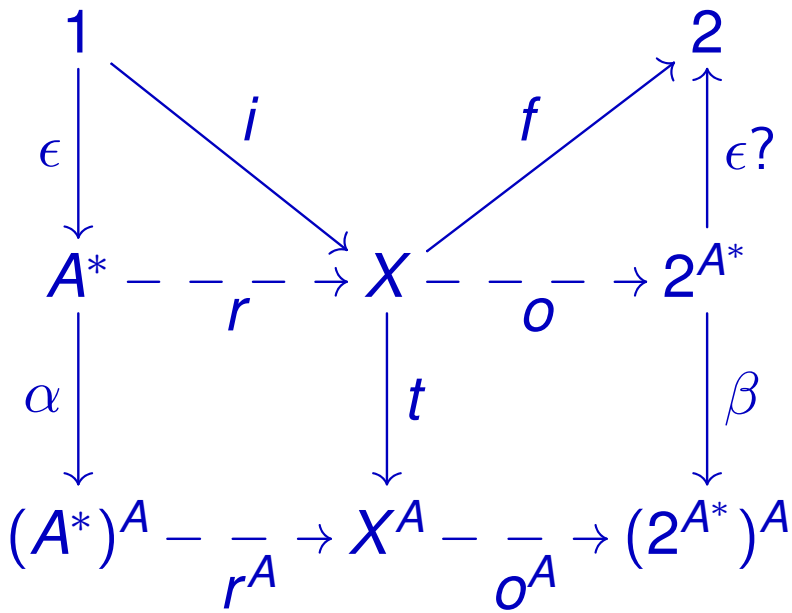


Back to today's picture



On the right: final coalgebra

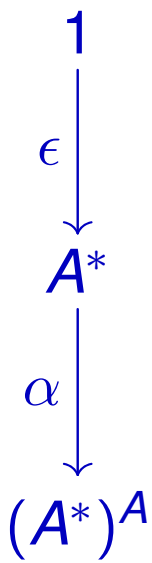
Back to today's picture



On the right: final coalgebra

On the left: initial algebra . . .

The “automaton” of words

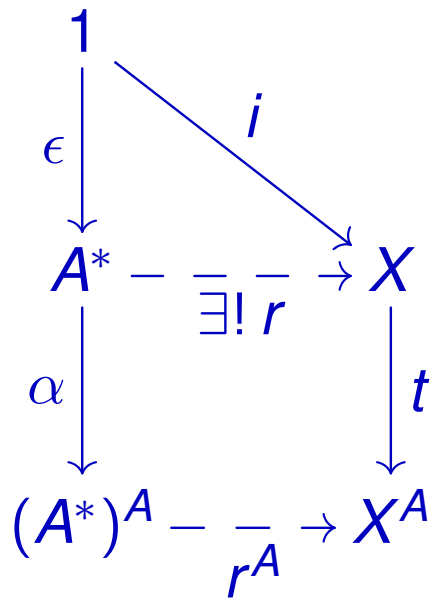


ϵ is initial state

$\alpha(w)(a) = w \cdot a$

that is, transitions: $w \xrightarrow{a} w \cdot a$

The automaton of words is . . . initial



$$i \in X \quad = \quad \text{initial state (to be precise: } i(0))$$

$$r(w) \quad = \quad i_w \quad = \quad \text{the state reached from } i \text{ after inputting } w$$

- Proof: easy exercise.
- Proof: formally, because A^* is an initial $1 + A \times (-)$-algebra!

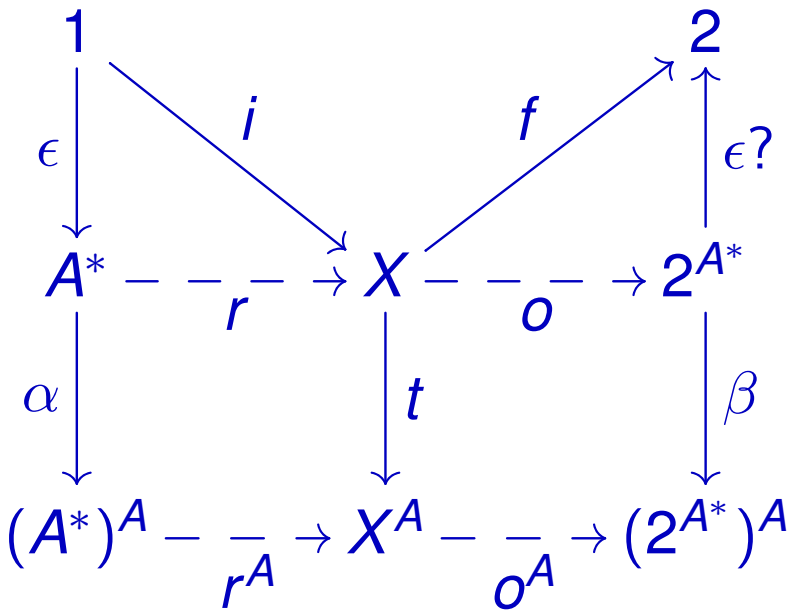
Duality

- Reachability and observability are dual:

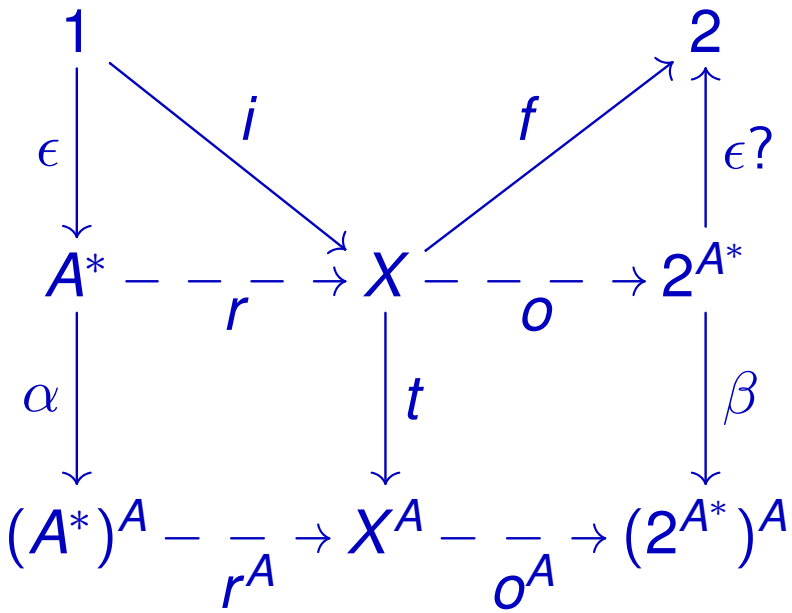
  Arbib and Manes, 1975.

- (here observable = minimal)

Reachability and observability



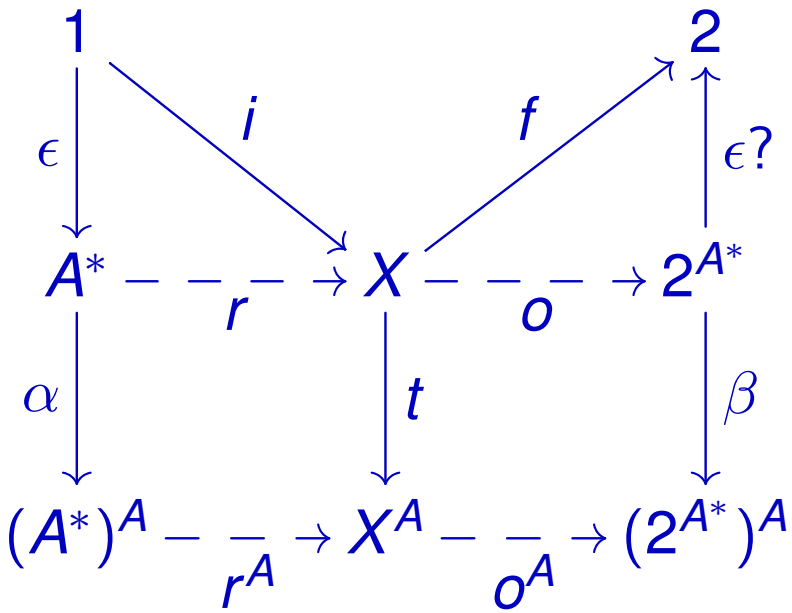
Reachability and observability



$r(w)$ = state reached on input w

$o(x)$ = language accepted by x

Reachability and observability

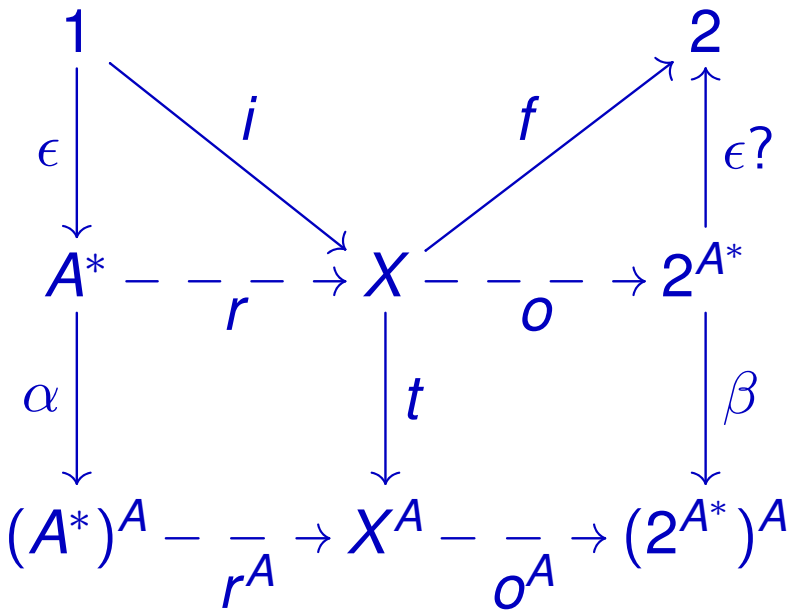


$r(w)$ = state reached on input w

$o(x)$ = language accepted by x

- We call X reachable if r is surjective.

Reachability and observability



$r(w)$ = state reached on input w

$o(x)$ = language accepted by x

- We call X reachable if r is surjective.
- We call X observable (= minimal) if o is injective.

Reversing the automaton

- Reachability $\leftrightarrow$ observability
- Being precise about homomorphisms is crucial.
- Forms the basis for proof Brzozowski's algorithm.

Powerset construction

$$2^{(-)} : \quad \begin{array}{c} V \\ \Big\downarrow g \\ W \end{array} \quad \mapsto \quad \begin{array}{c} 2^V \\ \Big\uparrow 2^g \\ 2^W \end{array}$$

Powerset construction

$$2^{(-)} : \quad \begin{array}{c} V \\ \Big\downarrow g \\ W \end{array} \quad \mapsto \quad \begin{array}{c} 2^V \\ \Big\uparrow 2^g \\ 2^W \end{array}$$

where $2^V = \{S \mid S \subseteq V\}$ and, for all $S \subseteq W$,

$$2^g(S) = g^{-1}(S) \quad (= \{v \in V \mid g(v) \in S\})$$

Powerset construction

$$2^{(-)} : \quad \begin{array}{c} V \\ \Big\downarrow g \\ W \end{array} \quad \mapsto \quad \begin{array}{c} 2^V \\ \Big\uparrow 2^g \\ 2^W \end{array}$$

where $2^V = \{S \mid S \subseteq V\}$ and, for all $S \subseteq W$,

$$2^g(S) = g^{-1}(S) \quad (= \{v \in V \mid g(v) \in S\})$$

- This construction is contravariant !!

Powerset construction

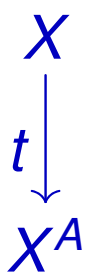
$$2^{(-)} : \quad \begin{array}{c} V \\ \Big\downarrow g \\ W \end{array} \quad \mapsto \quad \begin{array}{c} 2^V \\ \Big\uparrow 2^g \\ 2^W \end{array}$$

where $2^V = \{S \mid S \subseteq V\}$ and, for all $S \subseteq W$,

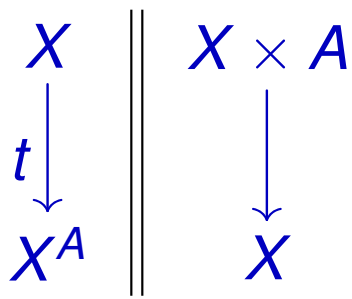
$$2^g(S) = g^{-1}(S) \quad (= \{v \in V \mid g(v) \in S\})$$

- This construction is contravariant !!
- Note: if g is surjective, then 2^g is injective.

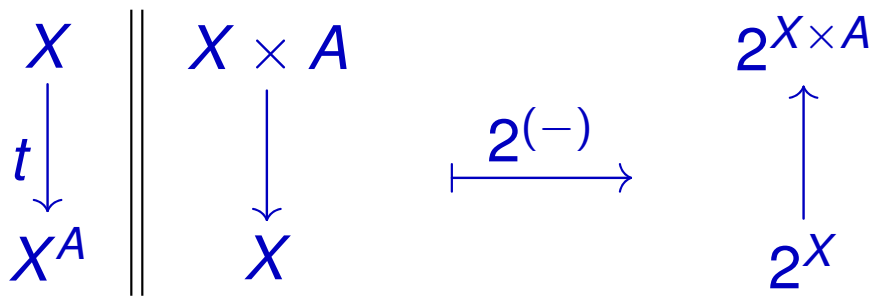
Reversing transitions



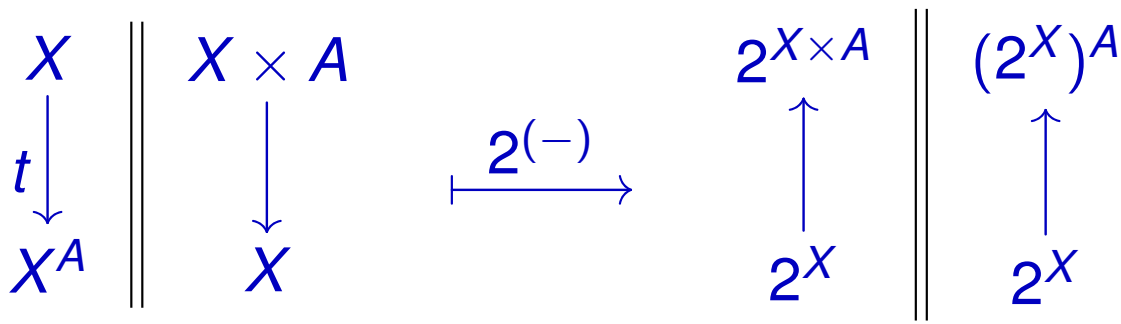
Reversing transitions



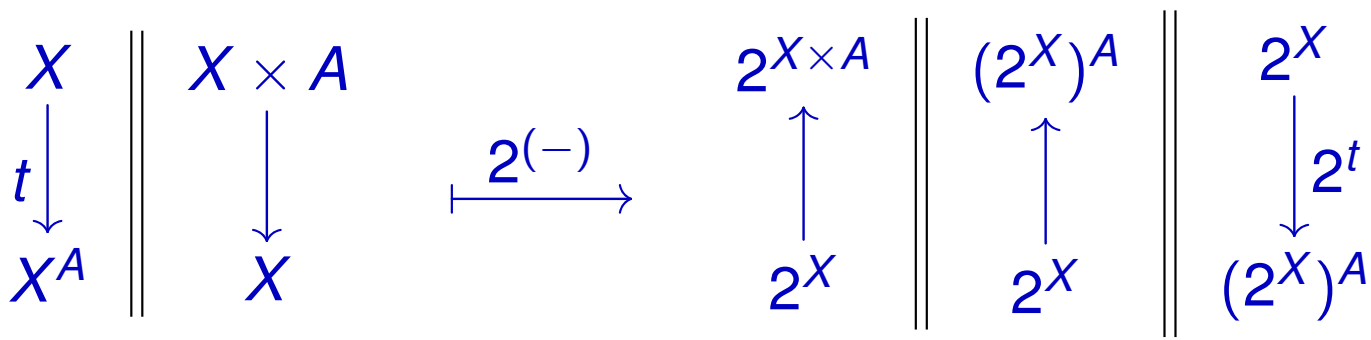
Reversing transitions



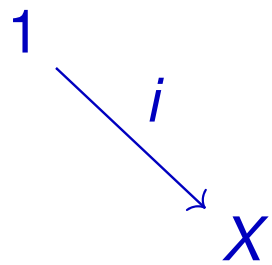
Reversing transitions



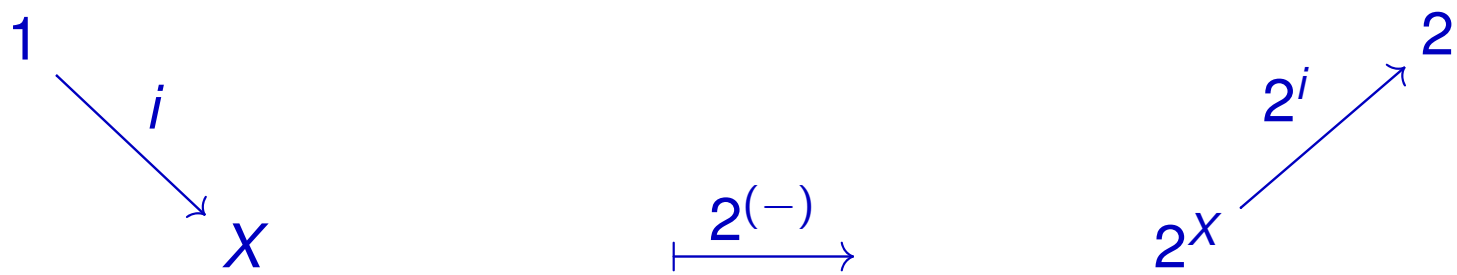
Reversing transitions



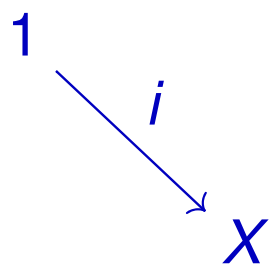
Initial $\leftrightarrow$ final

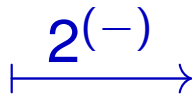


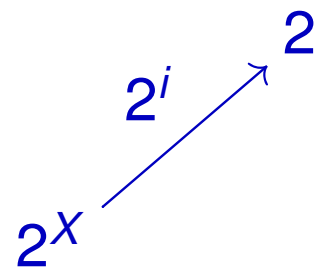
Initial $\leftrightarrow$ final

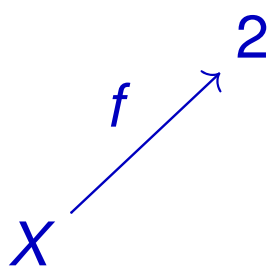


Initial $\leftrightarrow$ final

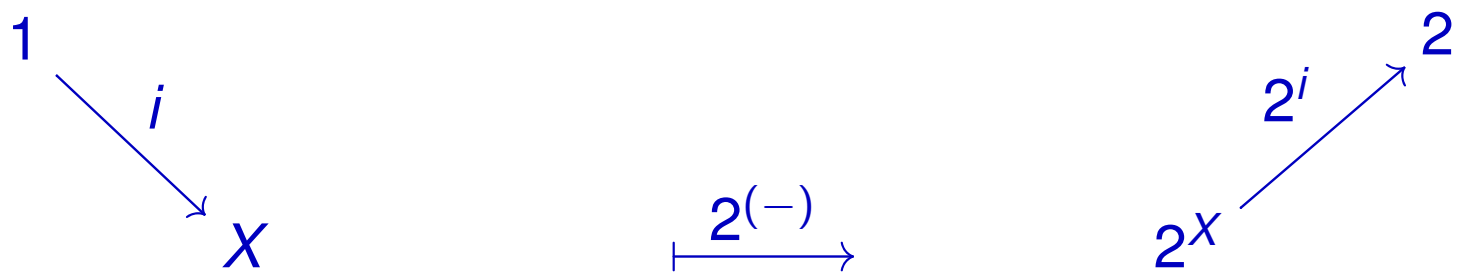


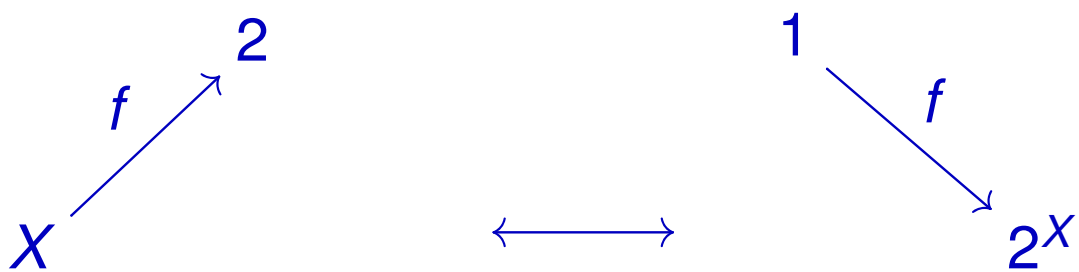




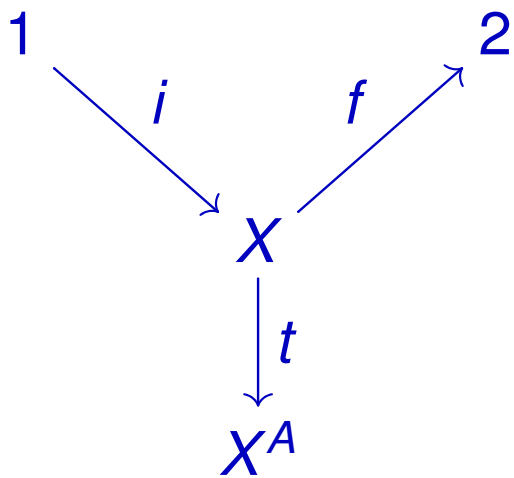


Initial $\leftrightarrow$ final

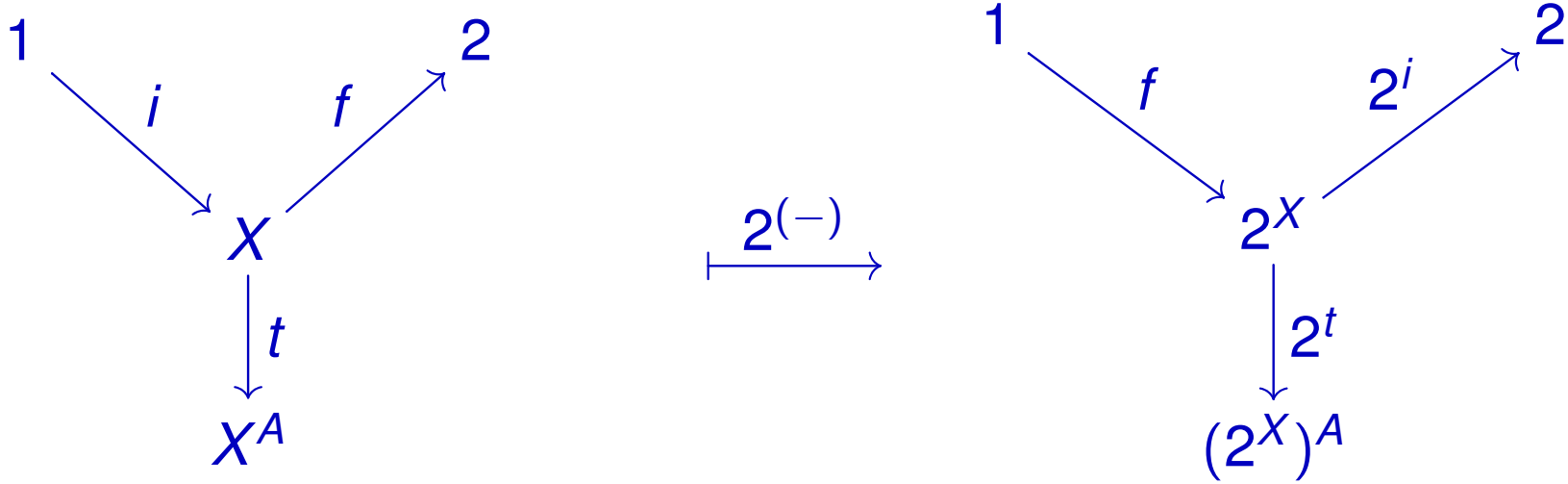




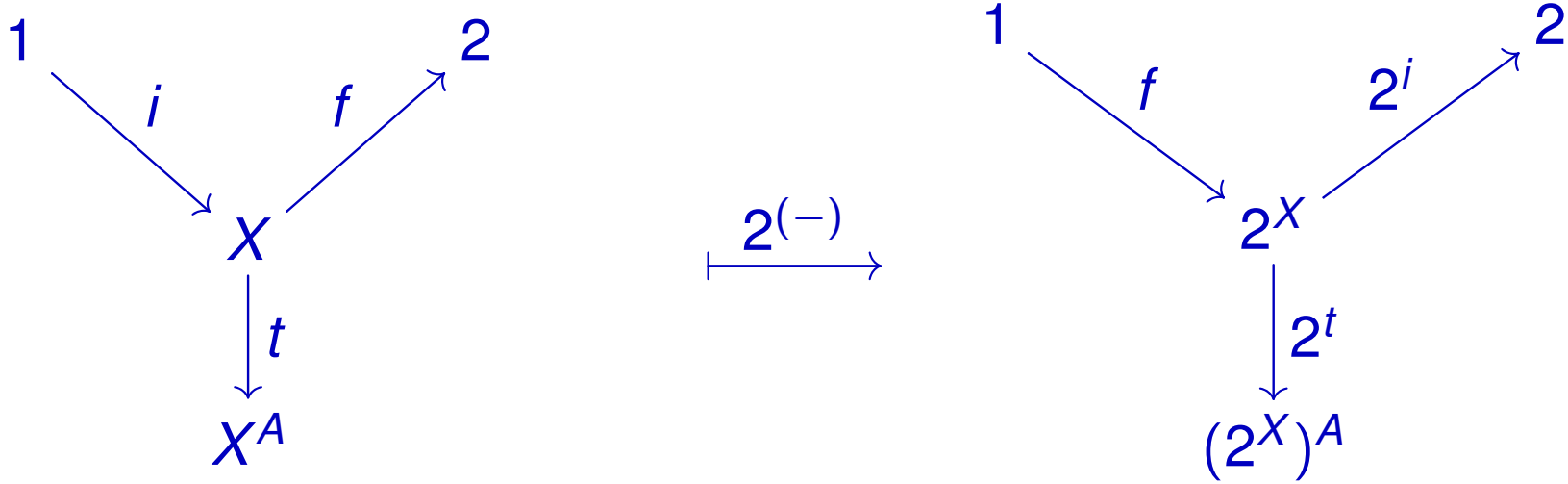
Reversing the entire automaton



Reversing the entire automaton

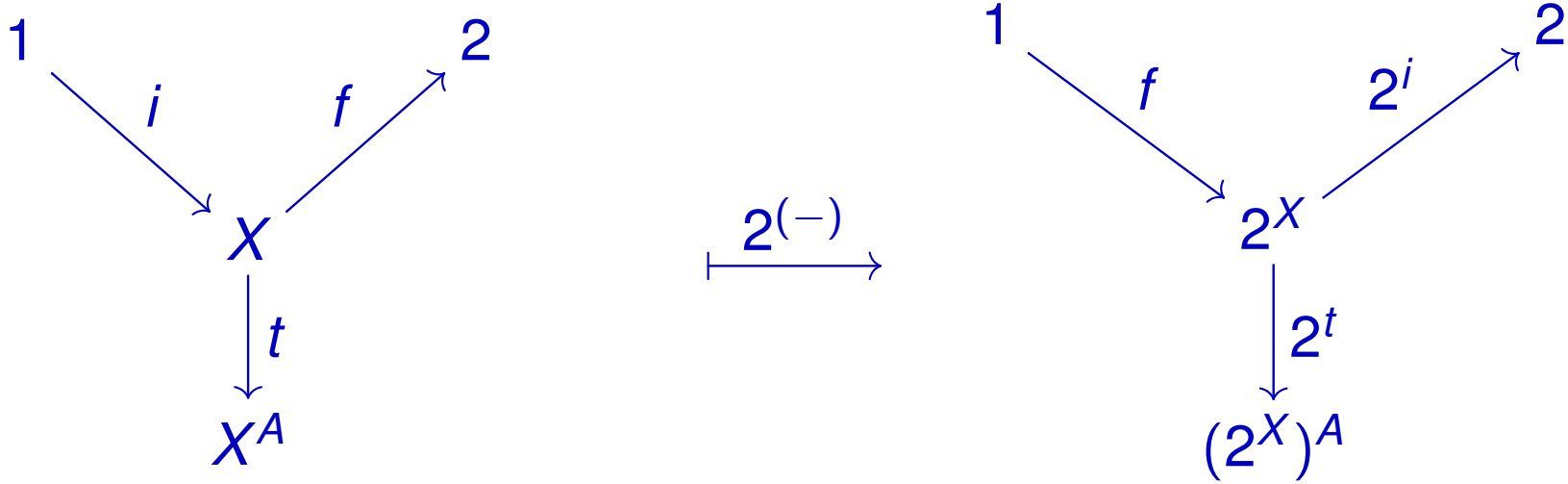


Reversing the entire automaton



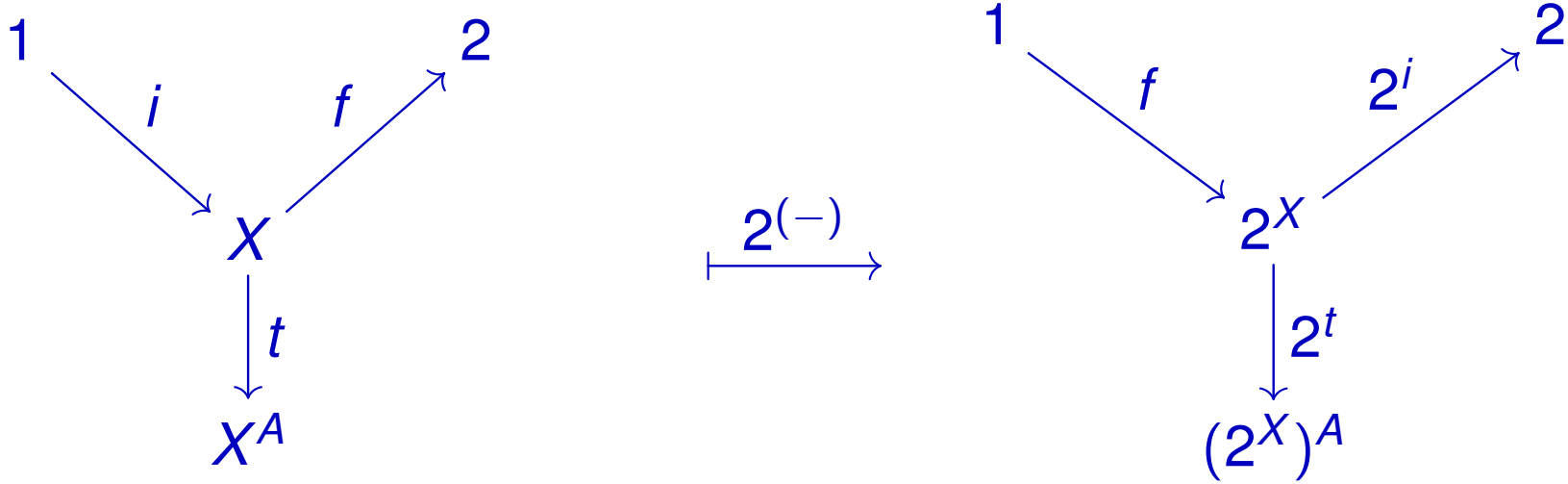
- Initial and final are exchanged . . .

Reversing the entire automaton



- Initial and final are exchanged . . .
- transitions are reversed . . .

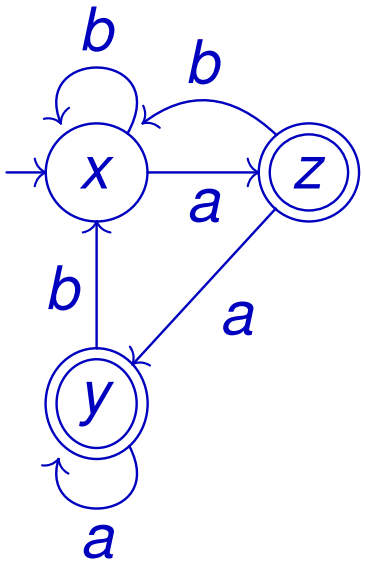
Reversing the entire automaton



- Initial and final are exchanged . . .
- transitions are reversed . . .
- and the result is again deterministic!

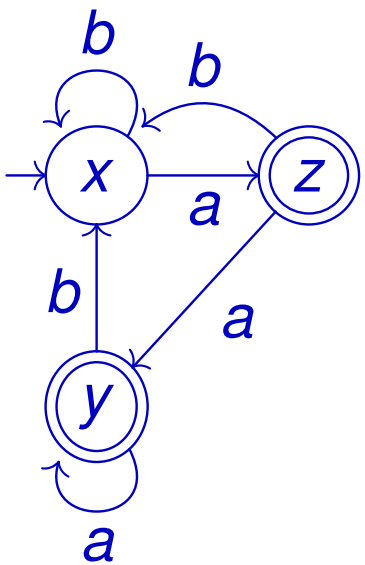
Our previous example

$X =$

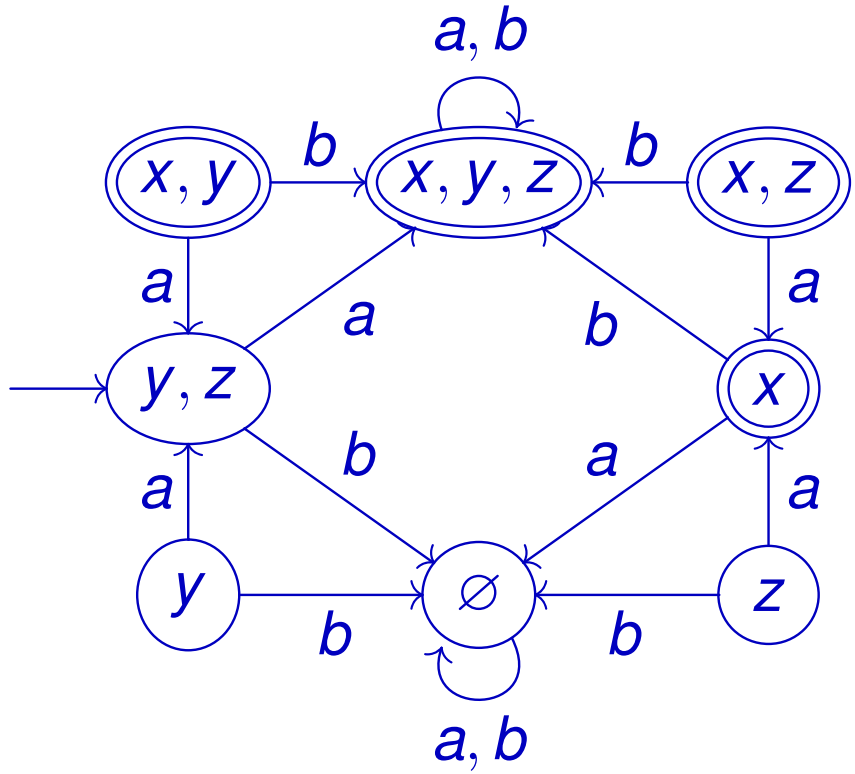


Our previous example

$X =$

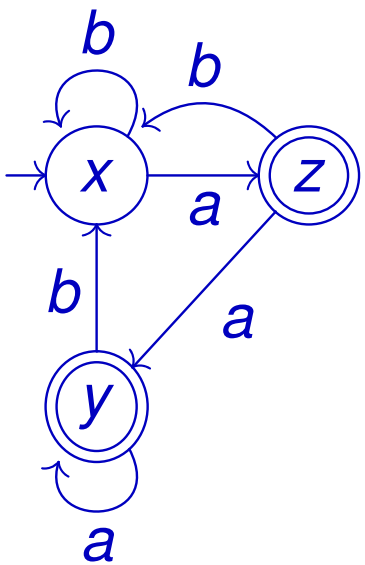


$2^X =$

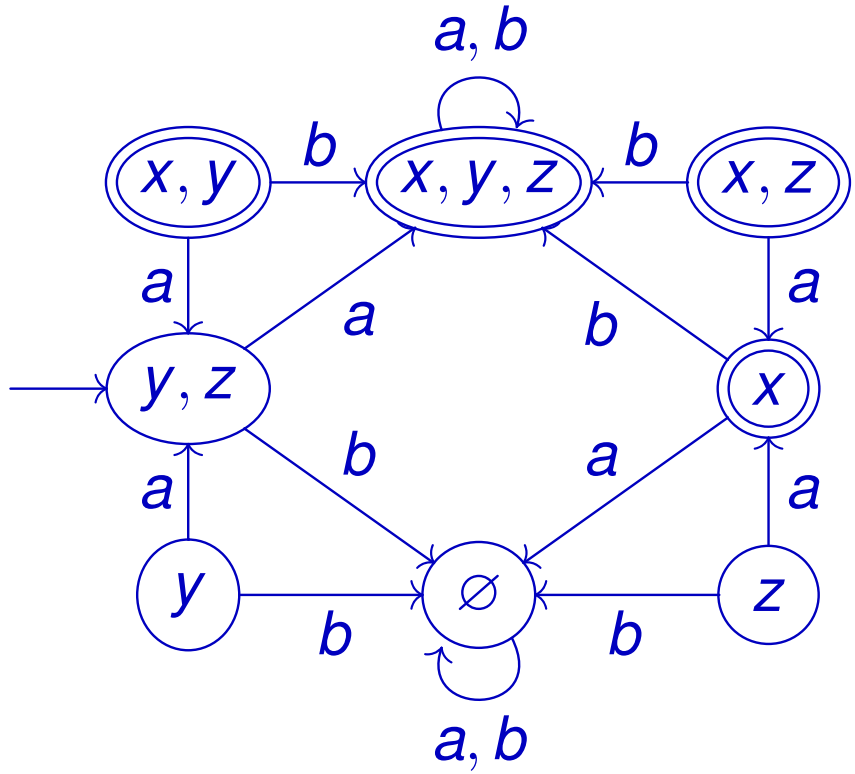


Our previous example

$X =$



$2^X =$



- Note that X has been reversed and determinized:

$$2^X = det(rev(X))$$

Proving today's Theorem

If: a deterministic automaton X is reachable and accepts $L(X)$

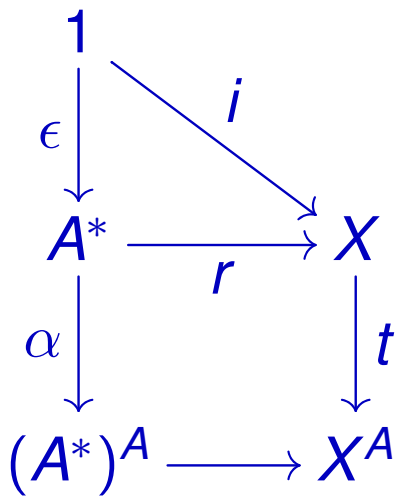
Proving today's Theorem

If: a deterministic automaton X is reachable and accepts $L(X)$

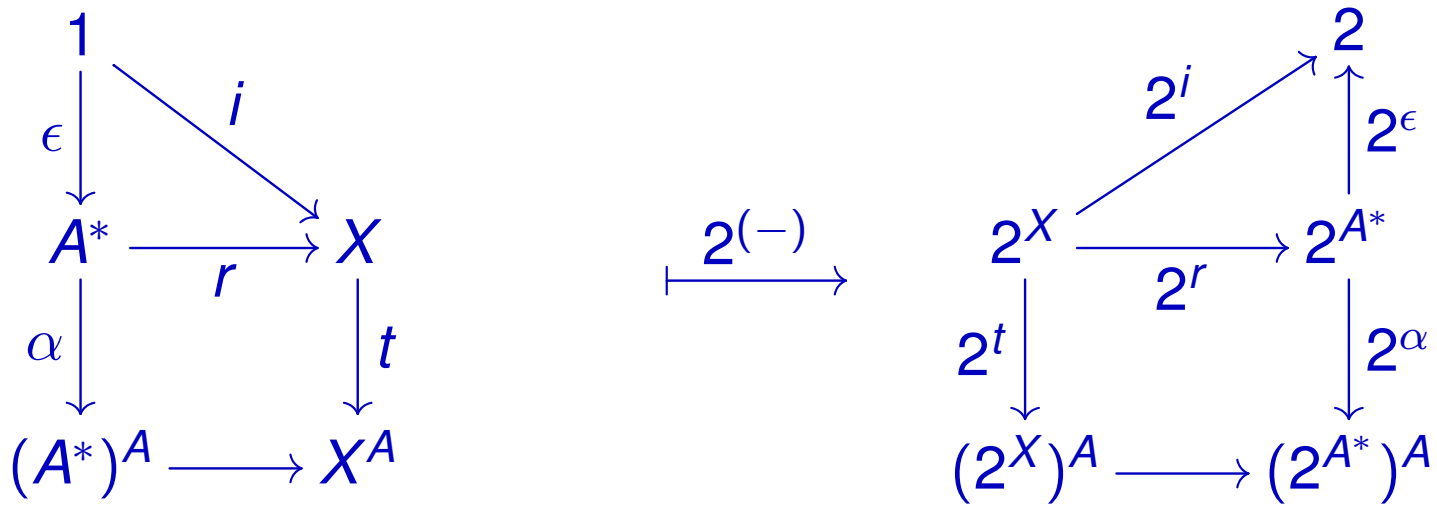
then: 2^X ($=$ $det(rev(X))$) is minimal/observable and

$$L(\,2^X\,) \;=\; reverse(\,L(X)\,)$$

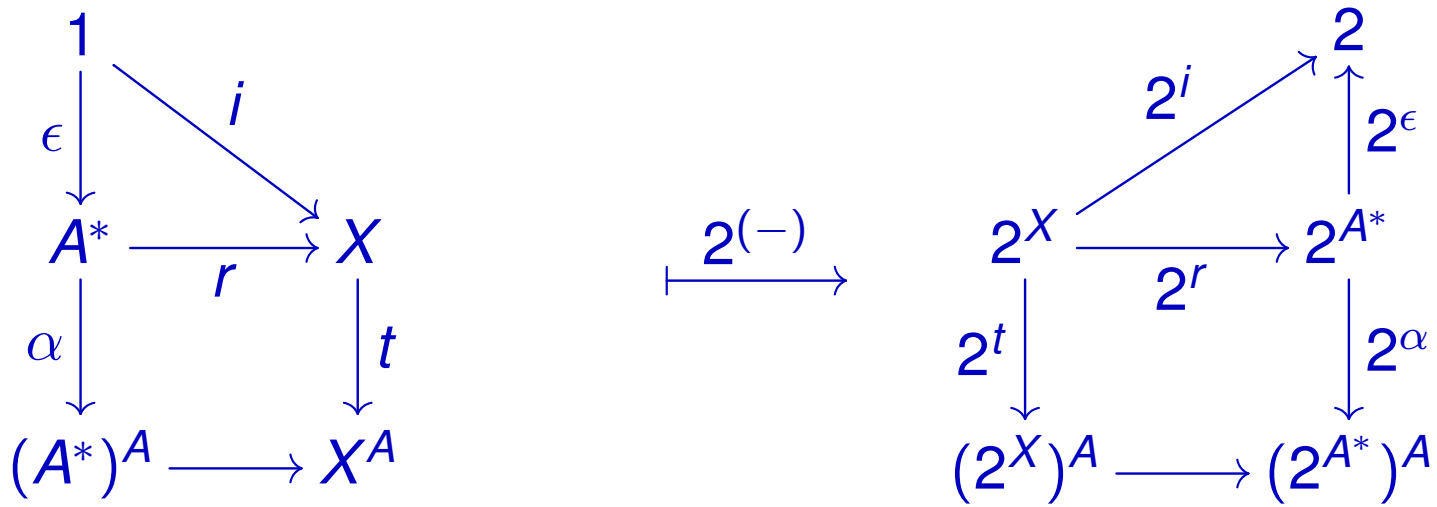
Proof: by reversing $A^* \xrightarrow{r} X$



Proof: by reversing $A^* \xrightarrow{r} X$

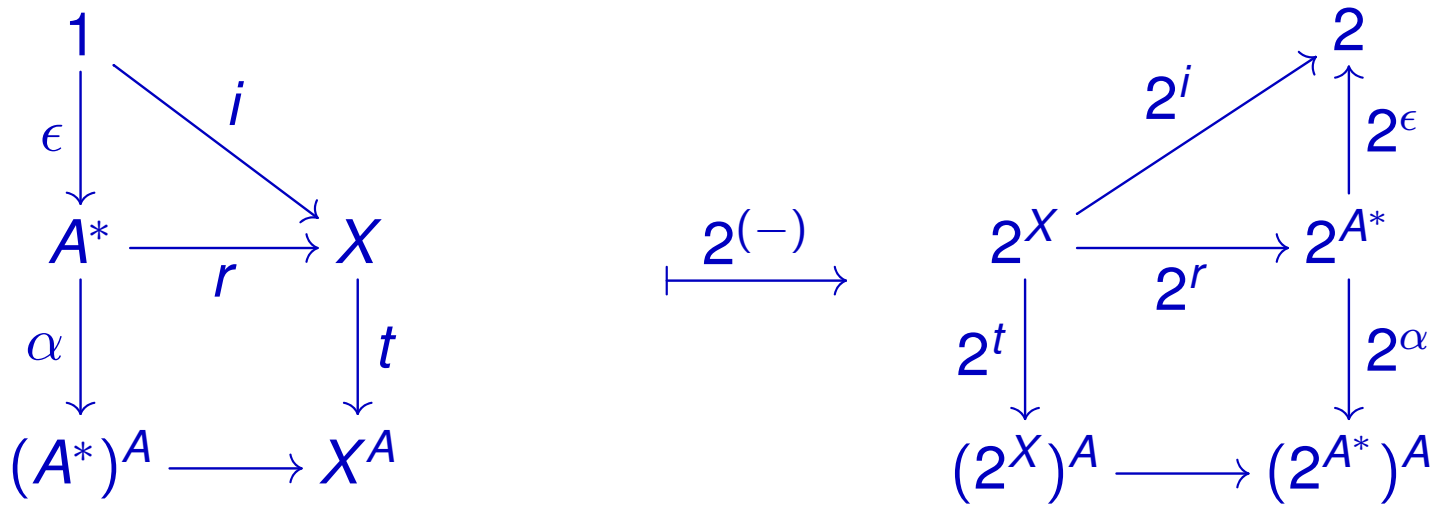


Proof: by reversing $A^* \xrightarrow{r} X$



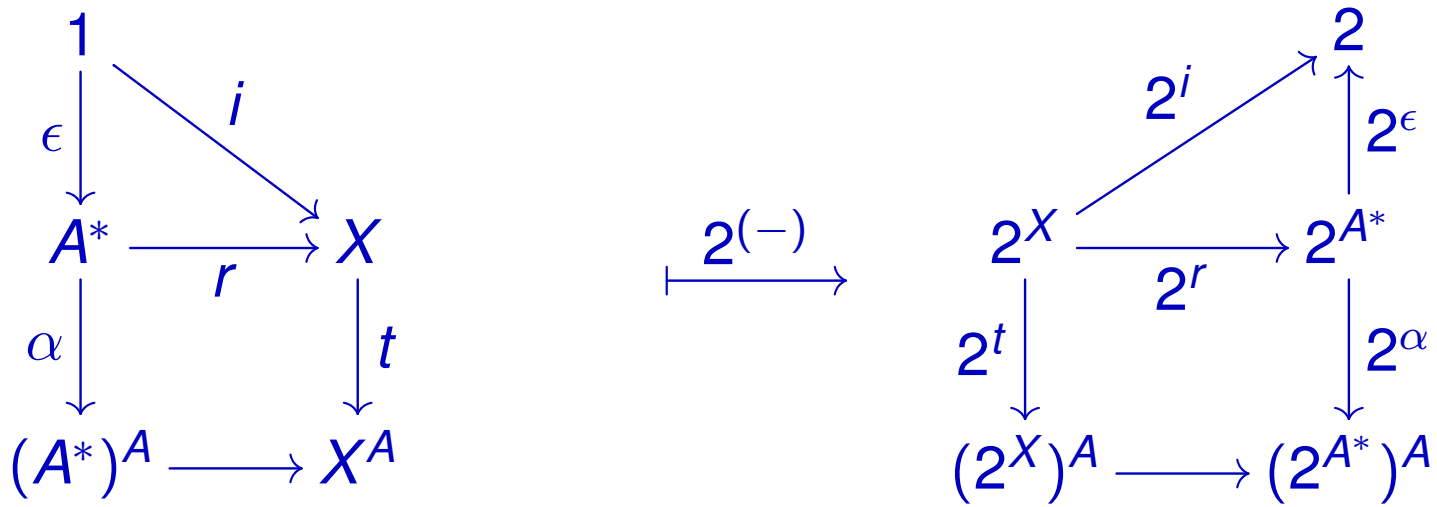
- X becomes 2^X

Proof: by reversing $A^* \xrightarrow{r} X$



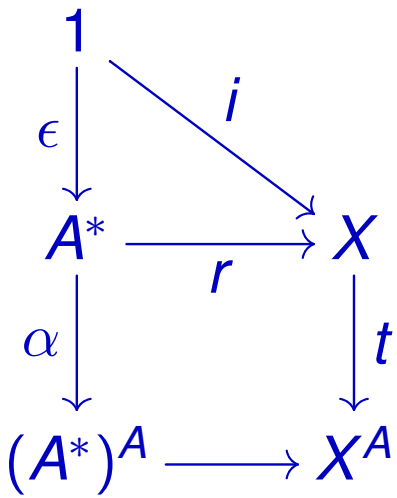
- X becomes 2^X
- initial automaton A^* becomes (almost) final automaton 2^{A^*}

Proof: by reversing $A^* \xrightarrow{r} X$

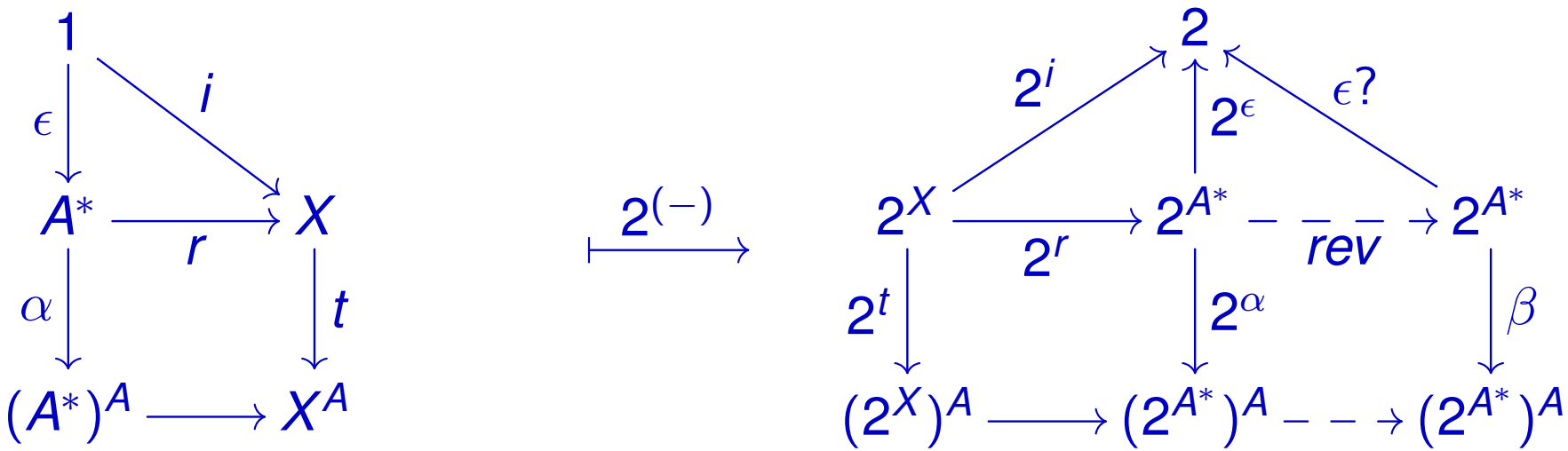


- X becomes 2^X
- initial automaton A^* becomes (almost) final automaton 2^{A^*}
- r is surjective $\Rightarrow$ 2^r is injective

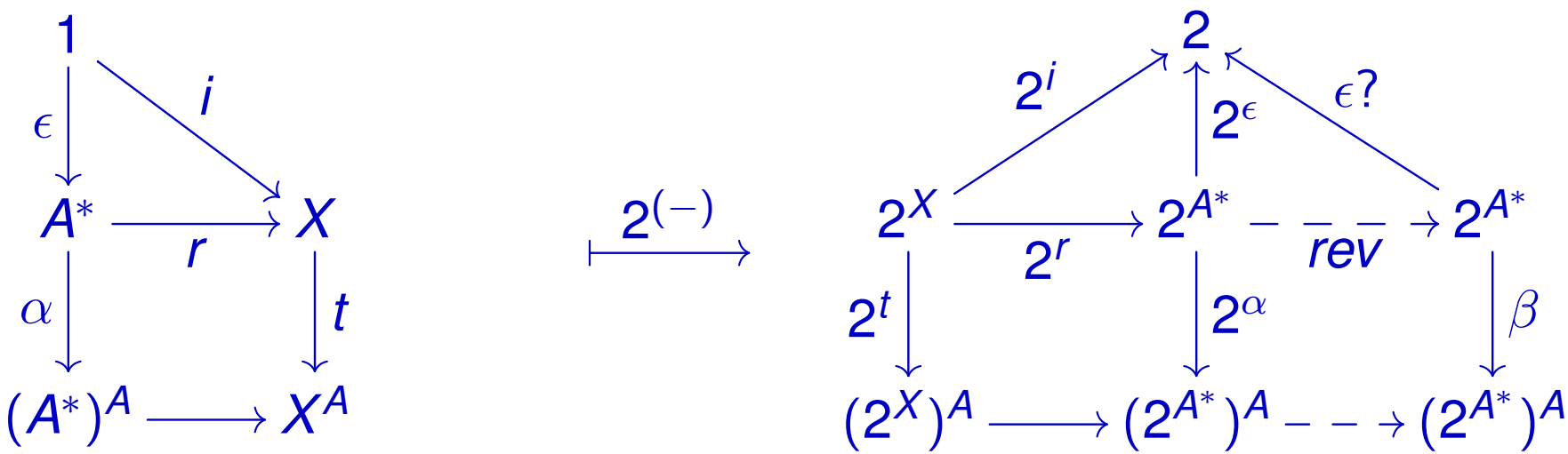
Reachable becomes observable



Reachable becomes observable

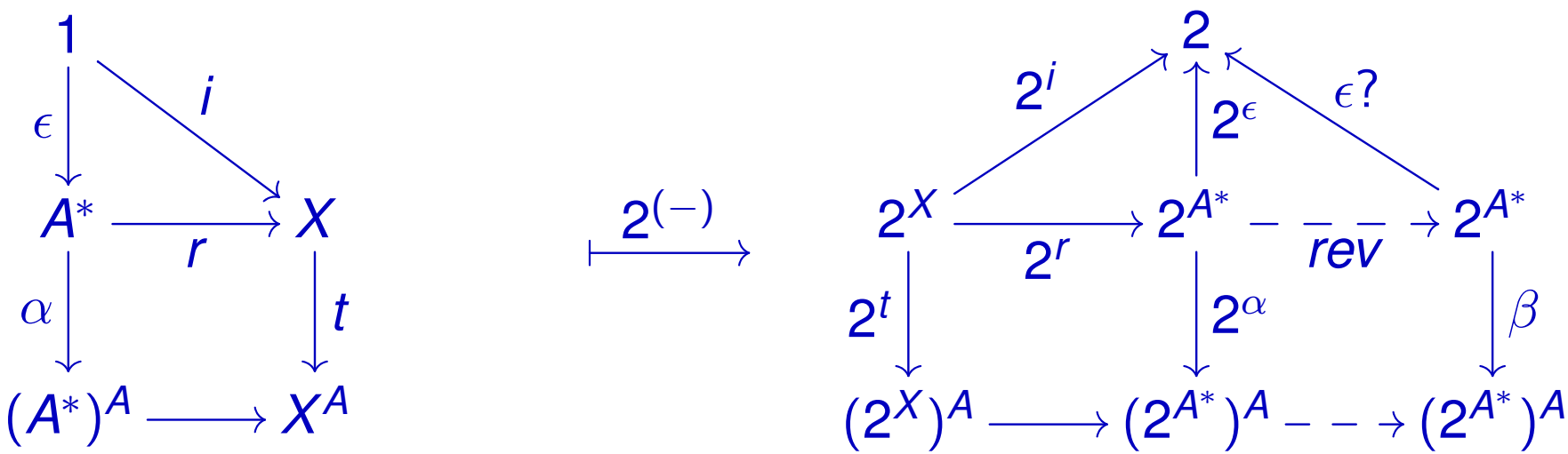


Reachable becomes observable



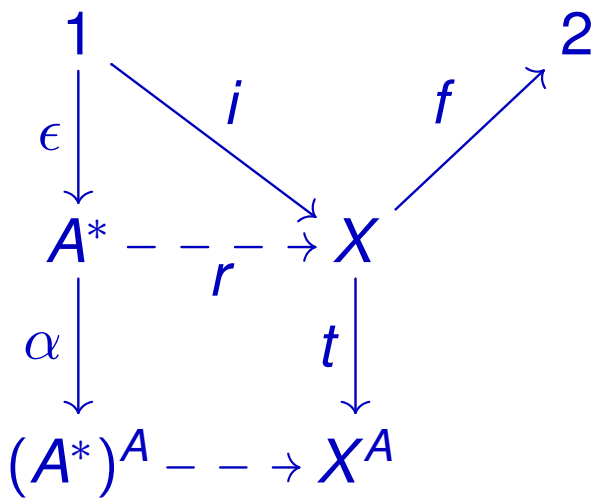
- If r is surjective then (2^r and hence) $rev \circ 2^r$ is injective.

Reachable becomes observable

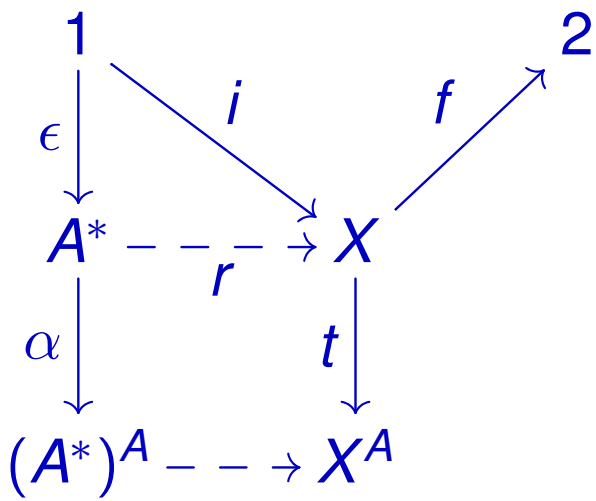


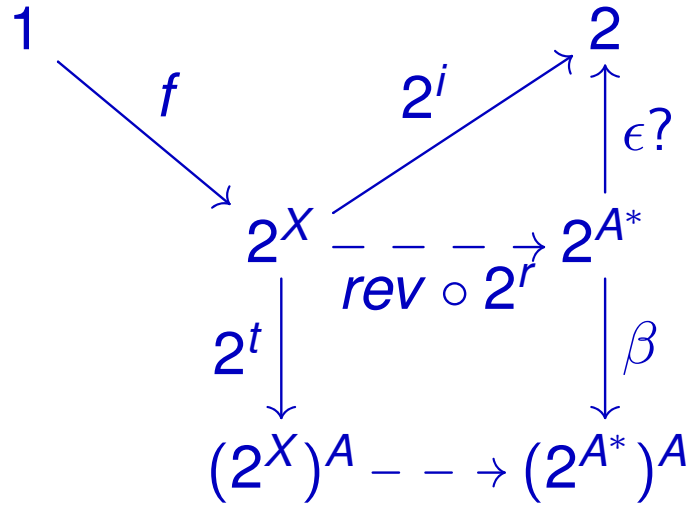
- If r is surjective then (2^r and hence) $rev \circ 2^r$ is injective.
- That is, 2^X is observable (= minimal).

Summarizing

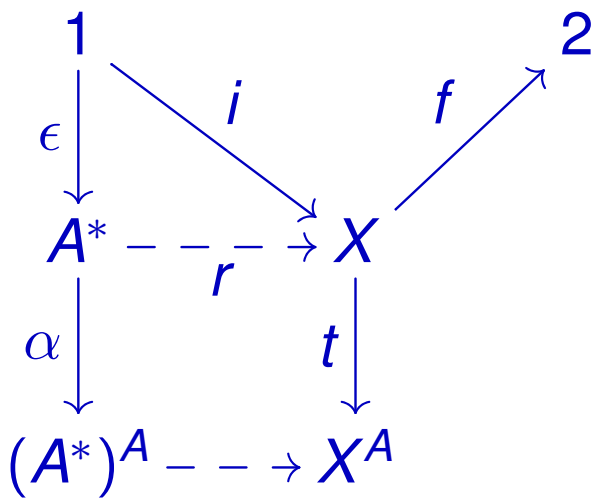


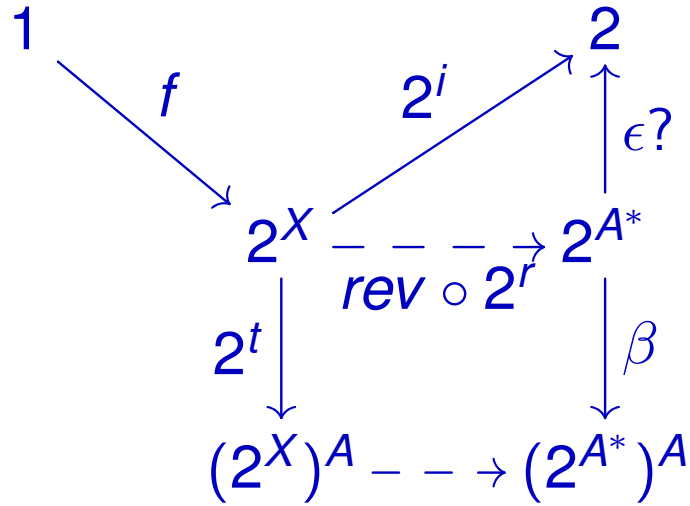
Summarizing





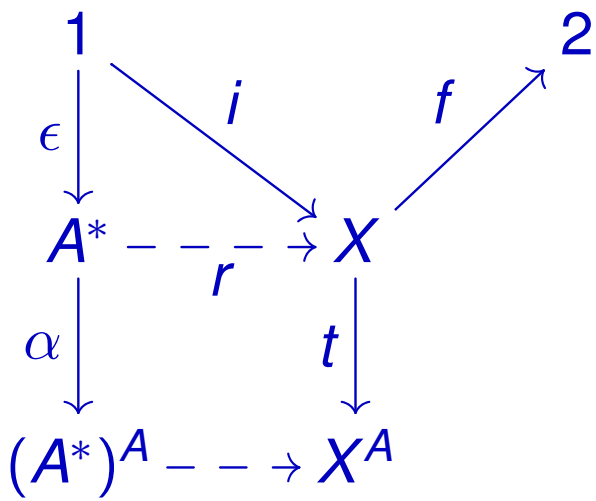
Summarizing

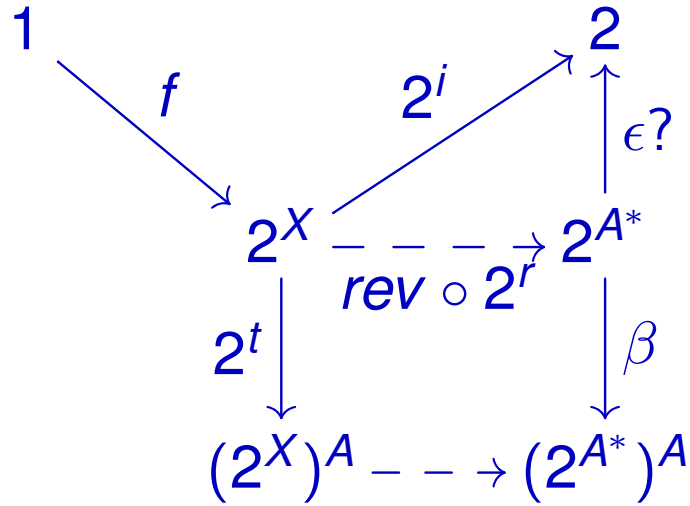




- If: X is reachable, i.e., r is surjective

Summarizing

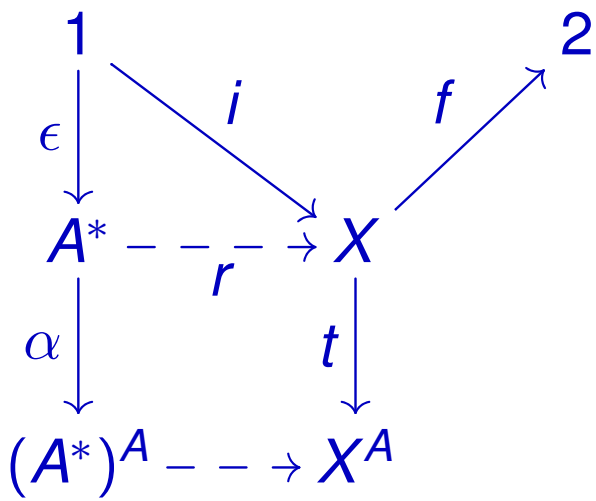


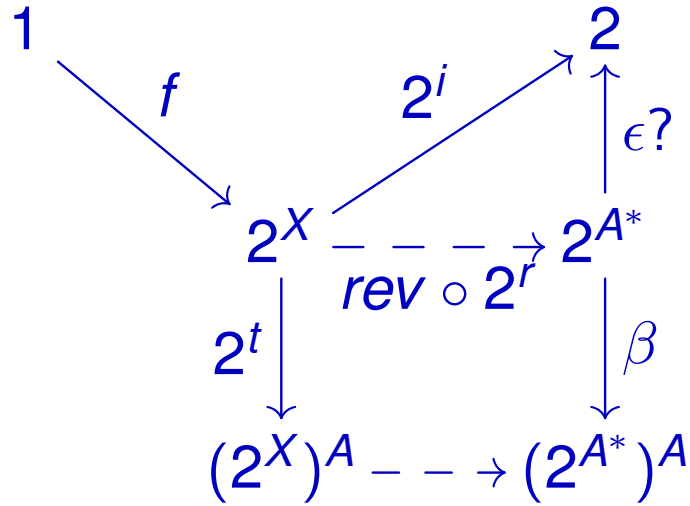


- If: X is reachable, i.e., r is surjective

  then: $rev \circ 2^r$ is injective, i.e., 2^X is observable = minimal.

Summarizing





- If: X is reachable, i.e., r is surjective

  then: $rev \circ 2^r$ is injective, i.e., 2^X is observable = minimal.
- And: $rev(2^r(f)) = rev(o(i))$, i.e., $L(\,2^X\,) \;=\; reverse(\,L(X)\,)$

Corollary: Brzozowski's algorithm

- X becomes 2^X, accepting $\mathit{reverse}(L(X))$

Corollary: Brzozowski's algorithm

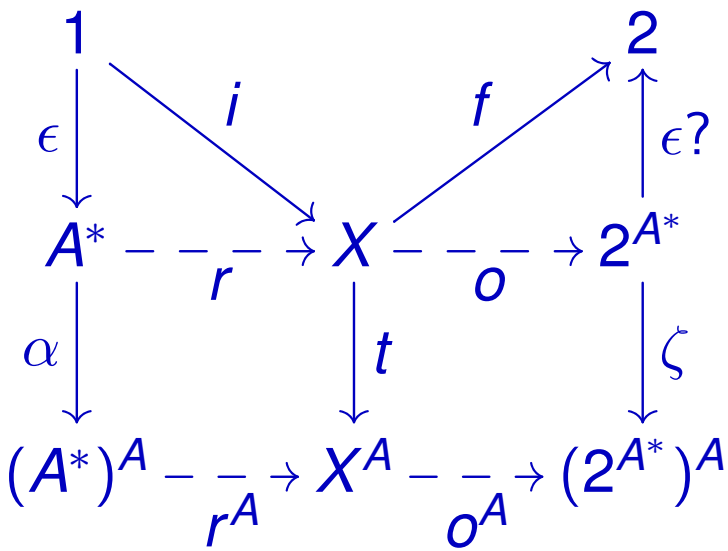
- X becomes 2^X, accepting $\mathit{reverse}(L(X))$
- take reachable part: $Y = \mathit{reachable}(2^X)$

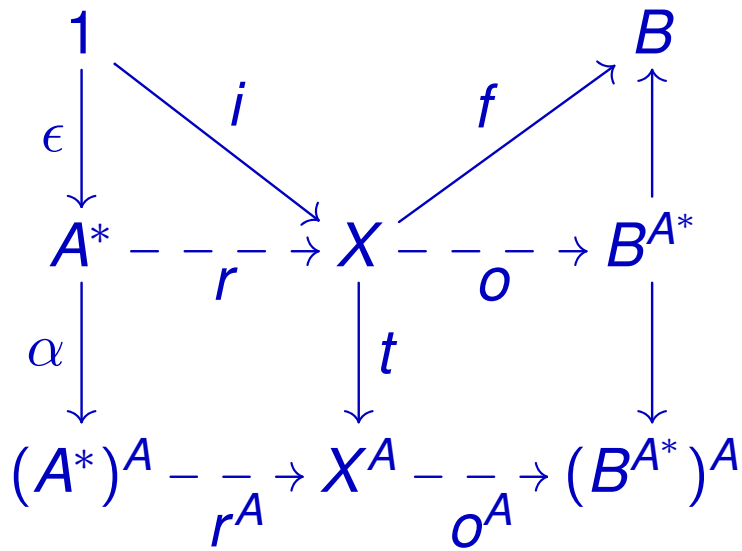
Corollary: Brzozowski's algorithm

- X becomes 2^X, accepting $\mathit{reverse}(L(X))$
- take reachable part: $Y = \mathit{reachable}(2^X)$
- Y becomes 2^Y, which is minimal and accepts

$$\mathit{reverse}(\mathit{reverse}(L(X))) = L(X)$$

Generalizations





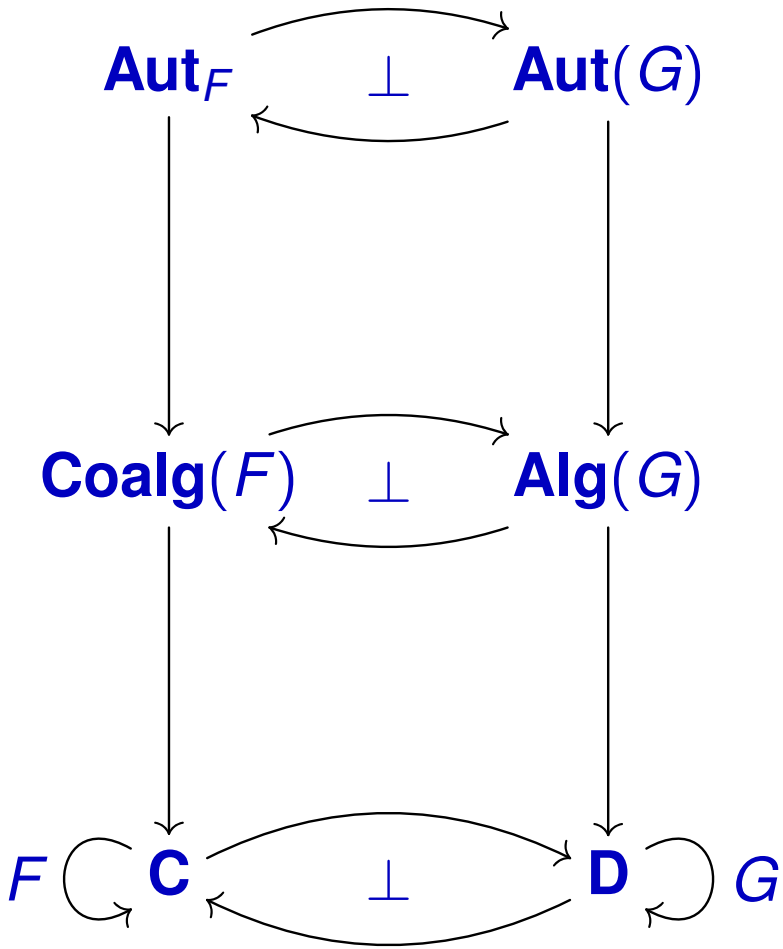
- A Brzozowski minimization algorithm for Moore automata.

$$B^X = \{\varphi \mid \varphi \colon X \to B\} \qquad B^f(\varphi) = \varphi \circ f$$

Further generalizations

- Moore automata generalization: uniform algorithm for decorated traces and must testing (joint work with Bonchi, Caltais and Pous);
- Further generalizations to non-deterministic and weighted automata.

A uniform picture based on duality



Conclusions

- Combination algebra-coalgebra is fruitful.
- Abstract analysis can bring new perspectives/results.
- (Co)algebra is not only semantics but also algorithms!

Conclusions

- Combination algebra-coalgebra is fruitful.
- Abstract analysis can bring new perspectives/results.
- (Co)algebra is not only semantics but also algorithms!

Thanks!