Towards a nominal Chomsky hierarchy

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Context

- Names are pervasive in computer science;
- Semantics of programming languages (α-equivalence);

$$f(a) = 2 * a \quad g(b) = 2 * b$$

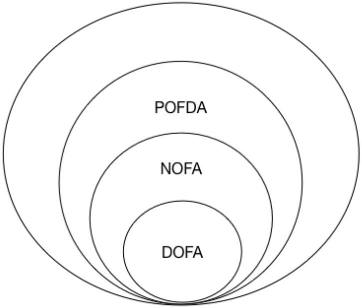
- Range of proposals for sound semantics:
  Pistore-Montanari, Gabbay-Pitts, ...
- Nominal sets (Fraenkel and Mostowski, early twentieth century).

Context

- Francez and Kaminski: finite memory automata.
- Montanari and Pistore: HD-automata.
- Murawski and Tzevelekos: fresh-register automata.
- Bojanczyk, Klin, Lasota: extensive results on nominal automata theory.
- Gabbay and Ciancia: nominal Kleene algebras.
- Kurz, Suzuki, Tuosto: regular expressions for HD-automata.

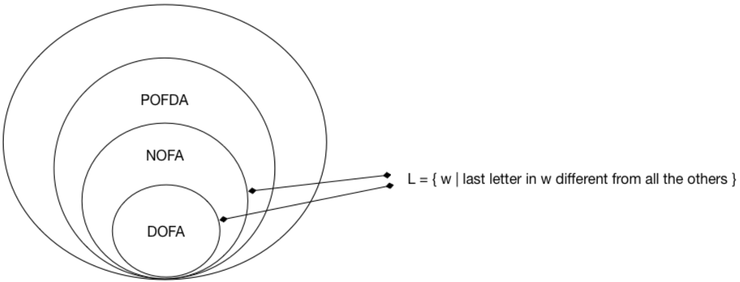
Nominal Chomsky hierarchy

Key point in Polish work: new notion of finiteness, orbit-finiteness.

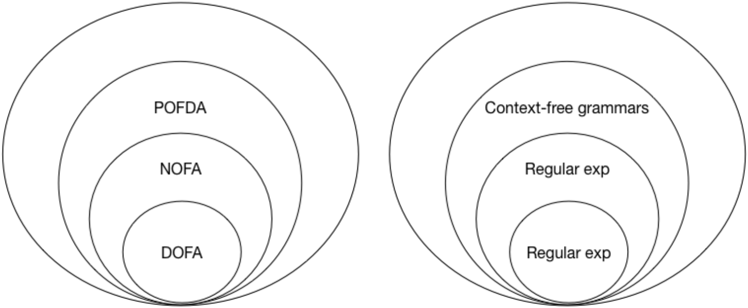


Nominal Chomsky hierarchy

Unexpected things happen with orbit-finiteness.

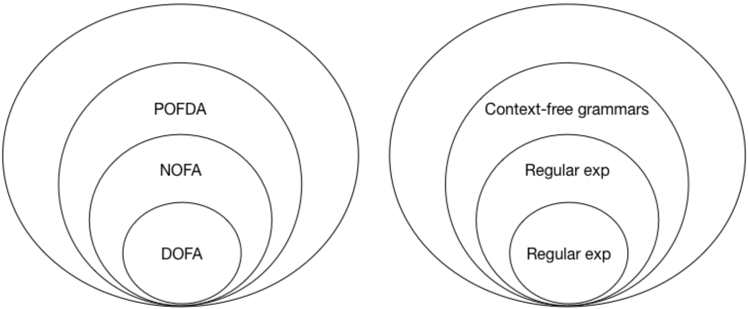


Nominal Chomsky hierarchy



Language hierarchy and correspondence theorems?

Nominal Chomsky hierarchy



Murawski (June 2015): to this day we still do not have a satisfactory notion of nominal regular language.

This talk: some results, many problems. . .

- Nominal Kleene algebra does not give a Kleene Theorem.
- New automaton model for one-sided Kleene Theorem.
- New proposal for regular languages.

Kozen, Mamouras, Silva. *Completeness and Incompleteness of Nominal KA.*

Kozen, Mamouras, Petrisan, Silva. *Nominal Kleene coalgebra.*

Nominal Sets [Gabbay & Pitts, LICS 1999]

Nominal Sets

- a convenient framework for name generation, binding, α-conversion

Applications

- logic: quantifiers
- programming language semantics: references, objects, pointers, function parameters
- XML document processing
- cryptography: nonces

Group Action

- Let G be a group and X a set
- A group action of G on X is a map $G \times X \to X$ such that

$$\pi(\rho x) = (\pi\rho)x \qquad\qquad 1x = x$$

- A G-set is a set X equipped with a group action $G \times X \to X$
- $f : X \to Y$ is equivariant if $f \circ \pi = \pi \circ f$ for all $\pi \in G$

Nominal Sets

- Let $\mathbb{A}$ be a countably infinite set of atoms
- Let G be the group of all finite permutations of $\mathbb{A}$ (permutations generated by transpositions (ab))
- If G acts on X, say that $A \subseteq \mathbb{A}$ supports $x \in X$ if

$$\mathrm{Fix}\, A \subseteq \mathrm{fix}\, x$$

where $\mathrm{fix}\, x = \{\pi \in G \mid \pi x = x\}$ and $\mathrm{Fix}\, A = \bigcap_{x \in A} \mathrm{fix}\, x$

Nominal Sets

- $x \in X$ has finite support if there is a finite $A \subseteq \mathbb{A}$ that supports x
- If $x \in X$ has finite support, then it has a minimum supporting set $\operatorname{supp} x$, the support of x
- Write $a \# x$ and say a is fresh for x if $a \notin \operatorname{supp} x$
- A nominal set is a set X with a group action of G such that every element has finite support

Nominal Sets

Example

- $\mathbb{A} = \{\text{variables}\}$
- $X = \{\lambda\text{-terms over } \mathbb{A}\}$
- If $\pi \in G$ and $\pi a = a$ for $a \in \mathrm{FV}(x)$, then $\pi x = x$ (α-conversion)
- $A \subseteq \mathbb{A}$ supports $x \iff \mathrm{FV}(x) \subseteq A$
- $\operatorname{supp} x = \mathrm{FV}(x)$
- $a \# x$ iff $a \notin \mathrm{FV}(x)$

$$(b\ c)\ ((\lambda b.a(bb))(\lambda b.a(bb))) = (\lambda c.a(cc))(\lambda c.a(cc))$$

More examples

- The set $\mathbb{A}$ is a G-set under the group action $\pi a = \pi(a)$. It is a nominal set with $\text{supp}(a) = \{a\}$.
- The set $\mathcal{P}\mathbb{A}$ is a G-set, but not a nominal set.
- The set $\mathcal{P}_{fs}\mathbb{A}$ of finite and co-finite subsets of $\mathbb{A}$ is a nominal set.

Kleene Algebra

Idempotent Semiring Axioms

$$p + (q + r) = (p + q) + r \qquad p(qr) = (pq)r$$
$$p + q = q + p \qquad 1p = p1 = p$$
$$p + 0 = p \qquad p0 = 0p = 0$$
$$p + p = p$$
$$p(q + r) = pq + pr \qquad a \leq b \overset{\triangle}{\iff} a + b = b$$
$$(p + q)r = pr + qr$$

Axioms for *

$$1 + pp^* \leq p^* \qquad q + px \leq x \;\Rightarrow\; p^*q \leq x$$
$$1 + p^*p \leq p^* \qquad q + xp \leq x \;\Rightarrow\; qp^* \leq x$$

Standard Model

Regular sets of strings over Σ

$$
\begin{aligned}
A + B &= A \cup B \\
AB &= \{xy \mid x \in A,\ y \in B\} \\
A^* &= \bigcup_{n \geq 0} A^n \quad = \quad A^0 \cup A^1 \cup A^2 \cup \cdots \\
1 &= \{\varepsilon\} \\
0 &= \emptyset
\end{aligned}
$$

This is the free KA on generators Σ

Relational Models

Binary relations on a set X

For $R, S \subseteq X \times X$,

$$
\begin{aligned}
R + S &= R \cup S \\
RS &= R \circ S \;=\; \{(u,v) \mid \exists w\ (u,w) \in R,\ (w,v) \in S\} \\
R^* &= \text{reflexive transitive closure of } R \\
&= \bigcup_{n \geq 0} R^n \;=\; R^0 \cup R^1 \cup R^2 \cup \cdots \\
1 &= \text{identity relation} \;=\; \{(u,u) \mid u \in X\} \\
0 &= \emptyset
\end{aligned}
$$

KA is complete for the equational theory of relational models

Other Models

- Trace models used in semantics
- (min, +) algebra used in shortest path algorithms
- (max, +) algebra used in coding
- Convex sets used in computational geometry (Iwano & Steiglitz 90)
- Matrix algebras

Nominal KA [Gabbay & Ciancia 2011]

A nominal Kleene algebra (NKA) over atoms $\mathbb{A}$ is a structure

$$(K, +, \cdot, ^*, 0, 1, \nu)$$

with $\nu : \mathbb{A} \times K \to K$ such that

- K is a nominal set over $\mathbb{A}$
- the KA operations and ν are equivariant:

$$\begin{aligned} \pi(x + y) &= \pi x + \pi y & \pi(0) &= 0 \\ \pi(xy) &= (\pi x)(\pi y) & \pi(1) &= 1 \\ \pi(x^*) &= (\pi x)^* & \pi(\nu a.e) &= \nu(\pi a).\pi e \end{aligned}$$

  equivalently, every $\pi \in G$ is an automorphism of K
- all the KA axioms are satisfied and ν satisfies...

Nominal Axioms [Gabbay & Ciancia 2011]

Odersky style axioms	interaction with KA operators
$a \# e \Rightarrow \nu a.e = e$	$\nu a.(d+e) = \nu a.d + \nu a.e$
$\nu a.\nu b.e = \nu b.\nu a.e$	$a \# e \Rightarrow (\nu a.d)e = \nu a.de$
$a \# e \Rightarrow \nu b.e = \nu a.(a\ b)e$	$a \# e \Rightarrow e(\nu a.d) = \nu a.ed$

Nominal KA [Gabbay & Ciancia 2011]

Expressions

$$e ::= a \in \Sigma \mid e + e \mid ee \mid e^* \mid 0 \mid 1 \mid \nu a.e$$

The operator νa is a binding operator whose scope is e

The set of expressions over Σ is denoted Exp_Σ

ν-strings

A ν-string is an expression with no occurrence of $+$, *, 0, or 1 (except to denote the null string, in which case we use ε)

$$x ::= a \in \Sigma \mid xx \mid \varepsilon \mid \nu a.x$$

The set of ν-strings over Σ is denoted Σ^ν.

Nominal Language Model [Gabbay & Ciancia 2011]

$$NL : \mathrm{Exp}_{\mathbb{A}} \to \mathcal{P}(\mathbb{A}^*)$$

Example:

$$NL(\nu a.ab) = \{ab \mid a \neq b\}$$

$$NL((\nu a.ab)(\nu a.ab)) = \{abcb \mid a, c \in \mathbb{A} \text{ distinct and different than } b\}$$

Care must be taken when defining product to avoid capture!

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Intermediate interpretation as sets of ν-strings over $\mathbb{A}$

$$I : \mathsf{Exp}_{\mathbb{A}} \to \mathcal{P}(\mathbb{A}^{\nu})$$

$+$, $\cdot$, *, 0, and 1 have their usual set-theoretic interpretations, and

$$I(\nu a.e) = \{\nu a.x \mid x \in I(e)\} \qquad I(a) = \{a\}.$$

Examples

$$I(\nu a.a) = \{\nu a.a\}$$
$$I(\nu a.\nu b.(a + b)) = \{\nu a.\nu b.a, \nu a.\nu b.b\}$$
$$I(\nu a.(\nu b.ab)(a + b)) = \{\nu a.(\nu b.ab)a, \nu a.(\nu b.ab)b\}$$
$$I(\nu a.(ab)^*) = \{\nu a.\varepsilon, \nu a.ab, \nu a.abab, \nu a.ababab, \ldots\}$$
$$I((\nu a.ab)^*) = \{\varepsilon, \nu a.ab, (\nu a.ab)(\nu a.ab), (\nu a.ab)(\nu a.ab)(\nu a.a$$

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I(\nu a.a) &= \{\nu a.a\} \\
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\end{aligned}$$

Nominal Language Model [Gabbay & Ciancia 2011]

$$NL : \mathbb{A}^{\nu} \to \mathcal{P}(\mathbb{A}^*)$$

- α-convert so that all bindings in x are distinct and different from free variables in x
- delete all binding operators νa to obtain $x' \in \mathbb{A}^*$
- $NL(x) = \{\pi(x') \mid \pi \in \text{fix}\,\text{FV}(x)\}$
- $NL(e) = \bigcup_{x \in I(e)} NL(x)$

Example

$$NL((\nu a.ab)(\nu a.ab)(\nu a.ab))$$
$$= \{abcbdb \mid a, c, d \in \mathbb{A} \text{ distinct and different from } b\}$$

Completeness and Incompleteness

Lemma

For $x, y \in \mathbb{A}^{\nu}$, $\vdash x = y$ if and only if $NL(x) = NL(y)$.

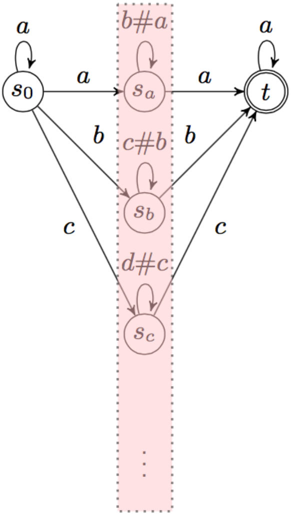
Incompleteness

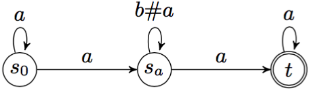
$\nvdash a \leq \nu a.a$ but $NL(a) = \{a\} \subseteq \mathbb{A} = NL(\nu a.a)$

Kleene Theorem

- Do we have a Kleene Theorem for NKA as in the classical case?
- Attempt 1: NKA denotes languages accepted by DOFA.

DOFA





- State space X – nominal set.
- Finite $\mapsto$ Orbit-finite

$$orbit(x) = \{\pi x \mid \pi \in G_{\mathbb{A}}\}$$

- Orbit-finite state space:
  $X = \{s_0\} + \mathbb{A} + \{t\}$
- Language accepted

$$\{w \in \mathbb{A}^* \mid \exists a.a \text{ occurs twice in } w\}$$

Kleene Theorem

NKA and the DOFA are equivalent?

- Unfortunately not...
- Simplest (worrying example): $(\nu a.a)^*$.

$$\{w \in \mathbb{A}^* \mid \text{all letters in } w \text{ are different}\}$$

- Not accepted by a deterministic or non-deterministic orbit-finite automaton!
- It is however accepted by a special *finite* nominal automaton

with a suitable acceptance condition.

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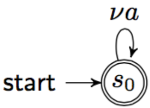


with a suitable acceptance condition.

A second attempt. . .

NKA and special nominal automata are equivalent.

- Unfortunately not. . .
- only one-side Kleene theorem.
- Special nominal automata are very similar to Tzevelekos fresh-register automata.

Nominal Kleene Coalgebra [ICALP'15]

- Nominal versions of the syntactic and semantic nominal Brzozowski derivative
- Finitely supported sets of ν-strings modulo the Gabbay–Ciancia axioms form the final coalgebra
- Half a Kleene theorem (expressions $\Rightarrow$ automata)
- exponential space decision procedure

Nominal fresh-register automata

$(X, \mathrm{obs}, \mathrm{cont}, \mathrm{cont}_\nu)$

- X is a nominal set
- Equivariant transitions

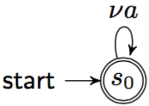
$$\begin{aligned} &\mathrm{obs} : X \to 2 \\ &\mathrm{cont}_a : X \to X,\ a \in \mathbb{A} \\ &\mathrm{cont}_{\nu a} : \{s \in X \mid a \# s\} \to X,\ a \in \mathbb{A} \end{aligned}$$

- Acceptor of ν-strings

$$\begin{aligned} \mathrm{Accept}(s, \varepsilon) &= \mathrm{obs}(s) \\ \mathrm{Accept}(s, am) &= \mathrm{Accept}(\mathrm{cont}_a(s), m) \\ \mathrm{Accept}(s, \nu a.am) &= \mathrm{Accept}(\mathrm{cont}_{\nu a}(s), m),\ a \# s. \end{aligned}$$

Example

The automaton



accepts exactly the ν-strings denoted by $(\nu a.a)^*$.

Crucial point: the automaton does not accept directly $w \in \mathbb{A}^*$.

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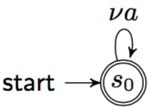

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Half of a Kleene Theorem

Theorem

Every expression corresponds to a nominal automaton.

Proof.

We show that an automaton structure can be defined inductively on the set of α-equivalence classes of NKA expressions $\mathrm{Exp}_{\mathbb{A}}/{\equiv_\alpha}$. □

Syntactic Brzozowski derivative

$$\mathsf{E}\colon \mathsf{Exp}_{\mathbb{A}}/{\equiv_\alpha} \to 2$$

$$\mathsf{E}(e_1 + e_2) = \mathsf{E}(e_1) + \mathsf{E}(e_2) \qquad \mathsf{E}(e_1 e_2) = \mathsf{E}(e_1)\mathsf{E}(e_2)$$

$$\mathsf{E}(a) = \mathsf{E}(0) = 0 \quad \mathsf{E}(1) = \mathsf{E}(e^*) = 1 \quad \mathsf{E}(\nu a.e) = \mathsf{E}(e)$$

Syntactic Brzozowski derivative

$$\mathrm{D}_a : \mathrm{Exp}_{\mathbb{A}}/\equiv_\alpha \to \mathrm{Exp}_{\mathbb{A}}/\equiv_\alpha \qquad \text{for } a \in \mathbb{A}.$$

$$\mathrm{D}_a(e_1 + e_2) = \mathrm{D}_a(e_1) + \mathrm{D}_a(e_2) \qquad \mathrm{D}_a(e_1 e_2) = \mathrm{D}_a(e_1)e_2 + \mathrm{E}(e_1)\mathrm{D}_a(e_2)$$

$$\mathrm{D}_a(e^*) = \mathrm{D}_a(e)e^* \qquad \mathrm{D}_a(0) = \mathrm{D}_a(1) = 0$$

$$\mathrm{D}_a(b) = \begin{cases} 1, & b = a \\ 0, & b \neq a \end{cases} \qquad \mathrm{D}_a(\nu b.e) = \begin{cases} 0, & b = a \\ \nu b.\mathrm{D}_a(e), & b \neq a \end{cases}$$

Syntactic Brzozowski derivative

$$\mathsf{D}_{\nu a} : \{e \in \mathsf{Exp}_{\mathbb{A}}/{\equiv_\alpha} \mid a\#e\} \to \mathsf{Exp}_{\mathbb{A}}/{\equiv_\alpha} \qquad \text{for } a \in \mathbb{A}.$$

$$\mathsf{D}_{\nu a}(e_1 + e_2) = \mathsf{D}_{\nu a}(e_1) + \mathsf{D}_{\nu a}(e_2)$$
$$\mathsf{D}_{\nu a}(e_1 e_2) = \mathsf{D}_{\nu a}(e_1)e_2 + \mathsf{E}(e_1)\mathsf{D}_{\nu a}(e_2)$$
$$\mathsf{D}_{\nu a}(e^*) = \mathsf{D}_{\nu a}(e)e^*$$
$$\mathsf{D}_{\nu a}(\nu b.e) = \nu b.\mathsf{D}_{\nu a}(e) + \mathsf{D}_a((a\ b)e),\ \ b \neq a$$
$$\mathsf{D}_{\nu a}(0) = \mathsf{D}_{\nu a}(1) = \mathsf{D}_{\nu a}(b) = 0$$

Example

For $b \neq a$,

1. $\mathsf{D}_{\nu a}(\nu b.bb) = \nu b.\mathsf{D}_{\nu a}(bb) + \mathsf{D}_a((a\ b)bb) = 0 + a = a$.
2. $\mathsf{D}_{\nu a}(\nu a.aa) = \mathsf{D}_{\nu a}(\nu b.bb) = a$.
3. $\mathsf{D}_{\nu a}(\nu a.ab) = \mathsf{D}_{\nu a}(\nu c.cb) = \nu c.\mathsf{D}_{\nu a}(cb) + \mathsf{D}_a(ab) = 0 + b = b$.
4. $\mathsf{D}_{\nu a}(\nu b.ba) = \nu b.\mathsf{D}_{\nu a}(ba) + \mathsf{D}_a((a\ b)ba) = 0 + b = b$.

Antimirov derivative

There is an analog of the Antimirov derivative for NKA of type

$$\mathcal{A} : \mathsf{Exp}_{\mathbb{A}} \to (\wp\,\mathsf{Exp}_{\mathbb{A}})^{\mathbb{A}+\mathbb{A}}$$

$$\mathcal{A}_a(e_1 + e_2) = \mathcal{A}_a(e_1) \cup \mathcal{A}_a(e_2)$$
$$\mathcal{A}_a(e_1 e_2) = \mathcal{A}_a(e_1)\{e_2\} \cup \mathsf{E}(e_1)\mathcal{A}_a(e_2)$$

Half Kleene

Theorem (Half Kleene)

For every NKA expression e, there is a coalgebra X with designated start state s such that $L_X(s) = L(e)$. *The coalgebra has an orbit-finite nondeterministic representation given by the Antimirov representation of the Brzozowski derivatives of e.*

Halk of Kleene Theorem does not work for deterministic automata!

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Brzozowski vs Antimirov

$$e = (\nu a.a)^*(\nu a.a(\nu b.b)^* a).$$

$$e \xrightarrow{\nu a} e + (\nu b.b)^* a$$

$$\xrightarrow{\nu b} e + (\nu b.b)^* b + (\nu b.b)^* a$$

$$\xrightarrow{\nu c} e + (\nu b.b)^* c + (\nu b.b)^* b + (\nu b.b)^* a \xrightarrow{\nu d} \cdots$$

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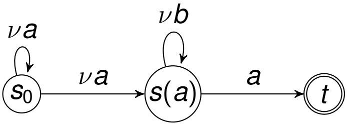

$$e \xrightarrow{\nu a} e + (\nu b.b)^* a$$

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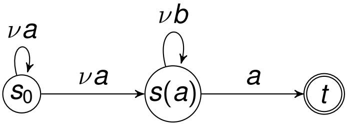

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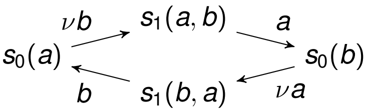
Open Problems

- Other half of the Kleene theorem is false:



The set of ν-strings accepted from state $s_0(a)$ is

$$\{\varepsilon,\ \nu b.ba,\ \nu b.ba(\nu a.ab),\ \nu b.ba(\nu a.ab(\nu b.ba)),\ \nu b.ba(\nu a.ab(\nu b.ba(\nu a.ab))), \ldots\}$$

Requires unbounded ν-depth!

Open Problems

- Can we characterize bounded ν-depth automata in a way that would lead to a converse of the Kleene theorem?
- Complexity?
- Can we extend the syntax of expressions to capture sets of unbounded ν-depth? Yes:

$$X_a = \varepsilon + \nu b.bY_{ab} \qquad\qquad Y_{ab} = aX_b$$

...but this leaves us with the task of providing proof rules and proving completeness (Nominal iteration algebras?)

Conclusions

- Nominal automata theory has important applications...
- There has been a lot of important work and progress...
- Unfortunately, there are still many open questions...
- ...and very basic ones.

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